LAND VALUE AND RENT DYNAMICS IN AN INTEGRATED WALRASIAN GENERAL EQUILIBRIUM AND NEOCLASSICAL GROWTH THEORY

Wei-Bin ZHANG[*]

Abstract

This paper is concerned with relationship between growth and land value change. It builds a heterogeneous-households growth model with endogenous wealth accumulation and fixed non-depreciating asset (land) in an integrated Walrasian general equilibrium and neoclassical growth theory. The production side consists of one service sector and one industrial sector. We use an alternative utility function proposed by Zhang, which enable us to develop a dynamic growth model with genuine heterogeneity. The wealth and income inequality is due to household heterogeneity in preferences and human capital as well as the households' initial wealth. This is different from the standard Ramsey-type heterogeneous-households growth models, for instance, by Turnovsky and Garcia-Penalosa (2008), where agents are heterogeneous only in their initial capital endowment, not in preference or/and human capital. We build a model for any number of types of household and provide a computational procedure for simulating model. For illustration we simulate the model for the economy with three types of households. We simulate the motion of the national economy and carry out comparative dynamic analysis. The comparative dynamic analysis provides some important insights. For instance, as the rich group increases its propensity to save, the GDP and land value are increased. In the long term the group accumulates more wealth, consumes more goods and services and accumulates more wealth. But in the long term the other two groups suffer from the rich households' preference change as their lot sizes, consumption levels of services and goods, and wealth are all reduced.

1. INTRODUCTION

A main concern of this study is dynamic interdependence between economic growth and land value change. Although issues related to land value change and economic structural change are significant, economics still needs an analytical framework for analyzing these issues. The literature on house and land prices has been increasingly expanding in recent years as surveyed by Cho (1996: 145), "During the past decade, the

[*] Ritsumeikan Asia Pacific University, Oita-ken, Japan, e-mail: *wbz1@apu.ac.jp*.

number of studies on intertemporal changes in house prices has increased rapidly because of wider availability of extensive micro-level data sets, improvements in modeling techniques, and expanded business applications." (e.g., Bryan and Colwell, 1982; Case and Quigley, 1991; Chinloy, 1992; Clapp and Giaccotto, 1994; Calhoun, et al. 1995; Quigley, 1995; Capozza and Seguin, 1996; Alpanda, 2012; Du and Peiser, 2014; Kok *et al.* 2014). Most of these studies are empirical. There are only a few formal growth models with endogenous land values. In his classical work *On the Principles of Political Economy and Taxation* of 1817, Ricardo tried to link wages, interest rate, and rent together in a compact theory. Ricardo distinguished between the three production factors, labor, capital, and land. He provided a theory to explain the functional income distribution of labor share, the capital, and the land rent share of total income. Ricardo (1821: preface) pointed out: "The produce … is divided among three classes of the commodity, namely, the proprietor of land, the owners of the stock or capital necessary for its cultivation, and laborers by whose industry it is cultivated. But in different stages of the society, the proportions of the whole produce of the earth which will be allotted to each of these classes, under the names of rent, profits, and wages, will be essentially different; depending mainly on the actual fertility of the soil, on the accumulation of capital and population, and on the skill, ingenuity, and the instruments in agriculture." Since the publication of the *Principles*, there are many studies which attempt to extend or generalize the Ricardian system (see Barkai, 1959, 1966; Pasinetti, 1960, 1974; Cochrane, 1970; Brems, 1970; Caravale and Tosato, 1980; Casarosa, 1985; Negish, 1989; Morishima, 1989). Nevertheless, as far as the current state of the literature is concerned, we can still apply what Ricardo (1821: preface) observed long time ago to describe the current situation: "To determine the laws which regulate this distribution, is the principal problem in Political Economy: much as the science has been improved by the writings of Turgot, Stuart, Smith, Say, Sismondi, and others, they afford very little satisfactory information respecting the natural course of rent, profit, and wages." In Ricardo's statement there is even no reference to land value (price). If land is owned by a single class and land owners' consumption behavior are neglected, it might be sufficient to deal with land rent. Nevertheless, in modern economies land is not owned by a single class. As households have different preferences and wealth levels and they might choose different bundles of assets in free markets, it is important to study land value. As recently reviewed by Liu *et al.* (2011: 1), "Although it is widely accepted that house prices could have an important influence on macroeconomic fluctuations, quantitative studies in a general equilibrium framework have been scant." This study makes a contribution to the literature by introducing endogenous land price into an integrated Walrasian general equilibrium and neoclassical growth theory recently proposed by Zhang (2012a).

Our model is based on the Walrasian general equilibrium theory of pure exchange and production economies. The theory is built on microeconomic foundation. It is proposed by Walras and further developed and refined by Arrow, Debreu and others in the 1950s (e.g., Walras, 1874; Arrow and Debreu, 1954; Gale, 1955; Nikaido, 1956, 1968; Debreu, 1959; McKenzie, 1959; Arrow and Hahn, 1971; Arrow, 1974; and Mas-Colell *et al.*, 1995). The theory is mainly concerned with market equilibrium with economic mechanisms of production, consumption, and exchanges with heterogeneous industries and households. The model in our study is Walrasian in the sense that for given levels of wealth there are competitive market equilibriums with heterogeneous industries and households. As the Walrasian general theory fails to be generalized and extended to growth theory of heterogeneous households with endogenous wealth (e.g., Morishima, 1964, 1977; Diewert,

1977; Eatwell, 1987; Dana et al. 1989; Jensen and Larsen, 2005; Montesano, 2008; Impicciatore et al., 2012), we apply neoclassical growth theory to introduce wealth accumulation of heterogeneous households. The neoclassical growth theory has been developed since the 1950s (e.g., Solow, 1956; Burmeister and Dobell, 1970; and Barro and Sala-i-Martin, 1995). Although this study follows Uzawa's two sector growth model in describing capital accumulation and economic structure (Uzawa, 1961; Stiglitz, 1967; Mino, 1996; Drugeon and Venditti, 2001; Jensen, 2003), we deviate from the traditional approach in modelling behaviour of households. This study examines land prices in a general equilibrium framework with homogeneous population and heterogeneous goods. The model in this study is based on the growth model with land and economic structure by Zhang (2012b, 2014). The main difference of this study from the previous models by Zhang is that this study assumes private landownership while Zhang's previous studies assume public landownership. The resulted dynamic models in the two approaches are different. This paper is organized as follows. Section 2 develops the growth model of wealth and income distribution with land distribution and housing. Section 3 examines dynamic properties of the model and simulates the model. Section 4 carries out comparative dynamic analysis. Section 5 concludes the study. The appendix proves the results in section 3.

2 THE GROWTH MODEL OF ECONOMIC STRUCTURE AND HETEROGENEOUS HOUSEHOLDS

We now build a general equilibrium dynamic growth model with heterogeneous households and land. The economy produces two goods: goods and services. We follow the neoclassical growth theory in describing production sectors. The core model in the neoclassical growth theory was the Solow one-sector growth model (Solow, 1956). As the one-sector model is not suitable for analyzing economic structural change and price changes of various goods, the Solow model has been extended to multiple sectors initially by Uzawa (1961, 1963), Meade (1961) and Kurz (1963). In the traditional two-sector economy, output of the capital sector is used entirely for investment and that of the consumption sector for consumption. Economists have generalized and extended the Uzawa two-sector model by, for instance, introducing more general production functions, more sectors, money, externalities, knowledge, human capital, and fictions in different markets (e.g., Takayama, 1985; Galor, 1992; Azariadis, 1993; Harrison, 2003; Cremers, 2006; Herrendorf and Valentinyi, 2006; Li and Lin, 2008; Stockman, 2009; Jensen and Lehmijoki, 2011; and Jensen *et al.*, 2001). Basing on the traditional approaches, this is concerned with the two-sector economy. We consider three inputs – labor, capital and land. Capital depreciates at a constant exponential rate, δ_k, which is independent of the manner of use. We use $r(t)$ to stand for the rate of interest. The households hold wealth and land and receive income from wages, land rent, and interest payments of wealth. Land is only for residential use and service supply. Technologies of the production sectors are characterized of constant returns to scale. All markets are perfectly competitive and capital and labor are completely mobile between the two sectors. The industrial production is the same as that in Solow's one-sector neoclassical growth model. It is a commodity used both for investment and consumption. The service sector supplies services, which is used for consumption. The total land L is homogenous and constant. The land is owned by households and is distributed between housing and service production in free land market. The assumption of fixed land is also a strict requirement. As observed by Glaeser, *et al.* (2005), land supply elasticity varies

substantially over space in the USA (see also, Davis and Heathcote, 2007). This study neglects possible changes in land supply. The population is classified into J groups, each group with fixed population, $\overline{N}_j$. Let N for the flow of labor services used at time t for production. We assume that labor is always fully employed. We have

$$N = \sum_{j=1}^{J} h_j \overline{N}_j \tag{1}$$

where h_j are the levels of human capital of group j.

The Industrial Sector

The industrial sector uses capital and labor as inputs. We use subscript index, i and s, to denote respectively the industrial and service sectors. Let $K_j(t)$ and $N_j(t)$ stand for the capital stocks and labor force employed by sector j, $j = i, s$, at time t. We use $F_j(t)$ to represent the output level of sector j. The production function of the industrial sector is

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \ \ \alpha_i, \beta_i > 0, \ \ \alpha_i + \beta_i = 1, \tag{2}$$

where A_i, α_i, and β_i are parameters. Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. The wage rate, $w(t)$, is determined in labor market. Hence, for any individual firm, $r(t)$ and $w(t)$ are given at any point in time. The industrial sector chooses $K_i(t)$ and $N_i(t)$ to maximize profits. The marginal conditions are

$$r(t) + \delta_k = \frac{\alpha_i F_i(t)}{K_i(t)}, \ \ w(t) = \frac{\beta_i F_i(t)}{N_i(t)}. \tag{3}$$

The wage rate of group j is

$$w_j(t) = h_j w(t)$$

Service sector

The service sector employs three inputs, capital $K_s(t)$, labor force $N_s(t)$, and land $L_s(t)$, to produce services. We specify the production function as

$$F_s(t) = A_s K_s^{\alpha_s}(t) N_s^{\beta_s}(t) L_s^{\gamma_s}(t), \ \ \alpha_s, \beta_s, \gamma_s > 0, \ \ \alpha_s + \beta_s + \gamma_s = 1, \tag{4}$$

where A_s, α_s, β_s, and γ_s are parameters. We use $p_s(t)$, and $R(t)$ to represent respectively the price of services and the land rent. The marginal conditions are

$$r(t) + \delta_k = \frac{\alpha_s\, p_s(t) F_s(t)}{K_s(t)}, \quad w(t) = \frac{\beta_s\, p_s(t) F_s(t)}{N_s(t)}, \quad R(t) = \frac{\gamma_s\, p_s(t) F_s(t)}{L_s(t)}. \quad (5)$$

Choice between wealth and land

It is necessary to determine land ownership structure. Land may be owned by different agents under various institutions. This study assumes that land is owned by households. There are different approaches with regard to determination of land prices and rents. For instance, in the literature of urban economics two types of land distribution are often assumed. The one is the so-called absentee landlord. Under this assumption the landlords spend their land incomes outside the economic system. The another type, for instance as accepted in Kanemoto (1980), assumes that the urban government rents the land from the landowners at certain rent and sublets it to households at the market rent, using the net revenue to subsidize city residents equally. In some approaches (Iacoviello, 2005; Iacoviello and Neri, 2010) households are assumed to be credit constrained and these households use land or houses as collateral to finance consumption expenditures. These models with credit-constrained households are used to explain positive co-movements between house prices and consumption expenditures (see also, Campbell and Mankiw, 1989; Zeldes, 1989; Case, *et al.*, 2005; Mian and Sufi, 2010; Oikarinen, 2014). Liu *et al.* (2011) assume that firms are credit constrained, instead of households. Firms finance investment spending by using land as a collateral asset. Land can be sold and bought in free markets without any friction and transaction costs. Land use will not waste land and land cannot regenerate itself. Households own land and physical wealth. We use $p_L(t)$ to denote the price of land. Consider now an investor with one unity of money. He can either invest in capital good thereby earning a profit equal to the net own-rate of return $r(t)$ or invest in land thereby earning a profit equal to the net own-rate of return $R(t)/p_L(t)$. As we assume capital and land markets to be at competitive equilibrium at any point in time, two options must yield equal returns, i.e.

$$\frac{R(t)}{p_L(t)} = r(t). \quad (6)$$

This equation enables us to determine choice between owning land and wealth. Indeed, this assumption is made under many strict conditions. For instance, we neglect any transaction costs and any time needed for buying and selling. Expectations on land are complicated. Equation (6) also implies perfect information and rational expectation.

Behavior of households

For simplicity, we use lot size to stand for housing. As argued, for instance, by Davis and Heathcote (2007), most of the fluctuations in house prices are driven by land price rather than by the cost of structures. Consumers decide consumption levels of goods and services, housing, and how much to save. This study uses the approach to consumers' behavior proposed by Zhang (1993). We denote respectively physical wealth by $\bar{k}_j(t)$ and

land $\bar{l}_j(t)$ owned by the representative household. The total value of wealth owned by the household $a_j(t)$ is the sum of the two assets

$$a_j(t) = \bar{k}_j(t) + p_L(t)\bar{l}_j(t). \tag{7}$$

Per capita current income from the interest payment $r(t)\bar{k}_j(t)$, the wage payment $w_j(t)$, and the land revenue $R(t)\bar{l}_j(t)$ is given by

$$y_j(t) = r(t)\bar{k}_j(t) + h_j w(t) + R(t)\bar{l}_j(t). \tag{8}$$

We call $y_j(t)$ the current income in the sense that it comes from consumers' wages and current earnings from ownership of wealth. In the Solow one-sector growth model it is assumed that a fixed proportion of the current income is saved for the future consumption. Nevertheless, the Solowian approach neglects possible effects of wealth on households. Moreover, the available expenditure that a household spends is not necessary less than the current income as assumed in the Solow model. When the current income is not sufficient for consuming, the household may spend the past saving. As there are no taxes, the representative household's capita disposable income is

$$\hat{y}_j(t) = y_j(t) + a_j(t). \tag{9}$$

The disposable income is used for saving and consumption.

The household spends the disposable income on the lot size, consumption of services, consumption of industrial goods, and saving. The budget constraint is

$$R_j(t)l_j(t) + p_s(t)c_{sj}(t) + c_{ij}(t) + s_j(t) = \hat{y}_j(t). \tag{10}$$

This equation implies that the household's disposable income is entirely distributed between the consumption and saving. The utility function, $U_j(t)$, of the household is dependent on $l_j(t)$, $c_{sj}(t)$, $c_{ij}(t)$ and $s_j(t)$ as follows

$$U_j(t) = \theta_j l_j^{\eta_{0j}}(t)c_{sj}^{\gamma_{0j}}(t)c_{ij}^{\xi_{0j}}(t)s_j^{\lambda_{0j}}, \quad \eta_{0j}, \gamma_{0j}, \xi_{0j}, \lambda_{0j} > 0,$$

in which η_{0j}, γ_{0j}, ξ_{0j}, and λ_{0j} are a typical household's utility elasticity of lot size, services, industrial goods, and saving. We call η_{0j}, γ_{0j}, ξ_{0j}, and λ_{0j} household $j's$ propensities to consume housing, to consume services, to consume industrial goods, and to hold wealth, respectively.

By the way we mention that there are some other studies which also deal with growth with endogenous wealth accumulation and heterogeneous households. For instance, Turnovsky and Garcia-Penalosa (2008) propose a model to examine the dynamics of the

distributions of wealth and income. Their model is developed in in a Ramsey model in which agents differ in their initial capital endowment. Their model assumes that the agent maximizes lifetime utility, which is a function of both consumption and the amount of leisure time as:

$$Max \int_0^\infty T_{hj}^\sigma(t) c_{ij}^\xi(t) e^{-\beta t}\, dt$$

The preference parameters σ, ξ, and β are the same for all types of the households. Accordingly, the so-called heterogeneous households in this approach are not heterogeneous in preference, but are different only in initial wealth. The identical preference among different types of households is "necessary" because of a well-known property of the Ramsey-type growth theory as described by Turnovsky and Garcia-Penalosa (2008), "Early work examining the evolution of the distribution of wealth in the Ramsey model assumed agents that differ in their rate of time preferences. In this framework, the most patient agent ends up holding all the capital in the long run…". This implies that if households are different in their time preferences, the entire wealth is held only by one household and the rest of the population has no wealth in the long term. We apply Zhang's approach to analyze dynamic behavior of heterogeneous households with wealth accumulation.

Maximizing $U_j(t)$ subject to the budget constraint (10) implies

$$l_j(t) = \frac{\eta_j \, \hat{y}_j(t)}{R(t)}, \quad c_{sj}(t) = \frac{\gamma_j \, \hat{y}_j(t)}{p_s(t)}, \quad c_{ij}(t) = \xi_j \, \hat{y}_j(t), \quad s_j(t) = \lambda_j \, \hat{y}_j(t) \qquad (11)$$

where

$$\gamma_j \equiv \rho_j \gamma_{0j}, \quad \xi_j \equiv \rho_j \xi_{0j}, \quad \lambda_j \equiv \rho_j \lambda_{0j}, \quad \rho_j \equiv \frac{1}{\eta_{0j} + \gamma_{0j} + \xi_{0j} + \lambda_{0j}}$$

According to the definition of $s_j(t)$, the change in wealth of the representative household from group j is

$$\dot{a}_j(t) = s_j(t) - a_j(t). \tag{12}$$

This equation simply implies that the change in wealth is the saving minus dissaving.

Balances of demand and supply for services
The demand and supply for the service sector's output balance at any point in time

$$\sum_{j=1}^{J} c_{sj}(t) \overline{N}_j = F_s(t) \tag{13}$$

All the land owned by households
The land owned by the population is equal to the national available land

$$\sum_{j=1}^{J} \bar{l}_j(t)\overline{N}_j = L. \tag{14}$$

Full employment of capital
We use $K(t)$ to stand for the total capital stock. We assume that the capital stock is fully employed. We have

$$K_i(t) + K_s(t) = K(t). \tag{15}$$

The value of physical wealth and capital
The value of physical capital is equal to the value of physical wealth

$$\sum_{j=1}^{J} \bar{k}_j(t)\overline{N}_j = K(t). \tag{16}$$

Full employment of labor force
We assume that labor force is fully employed

$$N_i(t) + N_s(t) = N. \tag{17}$$

The land market clearing condition implies
Land is used for the residential use and service production

$$\sum_{j=1}^{J} l_j(t)\overline{N}_j + L_s(t) = L. \tag{18}$$

We have thus built the model. It can be seen that the model is structurally a unification of the Walrasian general equilibrium and neoclassical growth theory with Zhang's approach to the household behavior. If we neglect the wealth accumulation and capital depreciation (i.e., capital being constant), then the model with heterogeneous households and multiple sectors belongs to the Walrasian general equilibrium theory. If we allow the households to be homogeneous, then the model is similar to the Uzawa model in the neoclassical growth theory. It should be noted that our model is not identical to the neoclassical growth theory. A main deviation from the traditional neoclassical growth theory is how to model behavior of households. We use an alternative utility function proposed by Zhang. What is important in this paper is that we include determination of land value and rent in a general equilibrium model with endogenous wealth accumulation.

3. THE DYNAMICS OF THE ECONOMY

The model has many variables and these variables are interrelated to each other in complicated ways. As there are different types of households and households have different propensities and human capital levels, the dynamics should be nonlinear and of high dimension. We now show that we can plot the motion of the system with initial conditions with computer. Before presenting the calculating procedure, we introduce

$$z(t) \equiv \frac{r(t) + \delta_k}{w(t)}$$

The following lemma gives a computational procedure for plotting the motion of the dynamic system.

Lemma

The motion of the economic system with J types of household is governed the following J nonlinear differential equations

$$\dot{z}(t) = \Omega_1\big(z(t),\ \{a_j(t)\}\big), \qquad\qquad \dot{a}_j(t) = \Omega_j\big(z(t),\ \{a_j(t)\}\big), \qquad (19)$$

where Ω_j are functions of $z(t)$ and $\{a_j(t)\} = (a_2(t),\ ...,\ a_J(t))$ given in the Appendix. Moreover, the values of all the other variables are determined uniquely as functions of $z(t)$ and $\{a_j(t)\}$ by the following procedure: $r(t)$ and $w(t)$ by (A2) $\rightarrow$ $a_1(t)$ by (A23) $\rightarrow$ $\hat{y}_j(t)$ by (A12) $\rightarrow$ $R(t)$ by (A16) $\rightarrow K(t)$ by (A19) $\rightarrow$ $K_i(t)$ and $K_s(t)$ by (A7) $\rightarrow$ $N_i(t)$ and $N_s(t)$ by (A1) $\rightarrow$ $p_L(t)$ by (6) $\rightarrow$ $L_s(t)$ by (A9) $\rightarrow$ $p_s(t)$ by (A5) $\rightarrow$ $F_i(t)$ by (2) $\rightarrow$ $F_s(t)$ by (4)$\rightarrow$ $c_{sj}(t),\ c_{ij}(t),\ l_j(t)$ and $s_j(t)$ by (11).

The lemma confirms that we have a set of nonlinear differential equations from which we can explicitly determine the motion of the J variables, $z(t)$ and $(a_2(t),\ ...,\ a_J(t))$. The dimension of the dynamic system is equal to the number of types of household. In a Walrasian general equilibrium theory where households are different from each other, the dimension of the dynamic system is the same as the population. As shown in the Appendix, we use $z(t)$ rather than $a_1(t)$ in the dynamic analysis as this enables us to find the set of differential equations by which we can solve the motion of all the variables by simulation. The computational procedure given in the lemma enables us to plot the motion of the economic system once the initial conditions are. Following the procedure with portable computer, we can illustrate the motion of the system. For simulation, we choose $J = 3$ and specify the parameter values

$$\overline{N}_1 = 5, \quad \overline{N}_2 = 10, \quad \overline{N}_3 = 20, \quad h_1 = 2, \quad h_2 = 1.5, \quad h_3 = 1, \quad A_i = 1.5, \quad A_s = 1, \quad \alpha_i = 0.3, \quad \alpha_s = 0.23,$$

$$\beta_s = 0.45, \quad L = 10, \quad \lambda_{01} = 0.7, \quad \xi_{01} = 0.1, \quad \gamma_{01} = 0.02, \quad \eta_{01} = 0.04, \quad \lambda_{02} = 0.65, \quad \xi_{02} = 0.11, \qquad (20)$$

$$\gamma_{02} = 0.02, \quad \eta_{02} = 0.04, \quad \lambda_{03} = 0.6, \quad \xi_{03} = 0.13, \quad \gamma_{03} = 0.02, \quad \eta_{03} = 0.04, \quad \delta_k = 0.05.$$

Group 1's, 2's and 3's population are respectively 5, 10 and 20. Group 1's, 2's and 3's level of human capital are respectively 2, 1.5 and 1 Group 1 (3) has the smallest (largest) population size and highest (lowest) human capital. The groups have also different preferences. The initial conditions are specified as

$$z(0) = 0.14, \quad a_2(0) = 12, \quad \overline{k}_3(0) = 6.2.$$

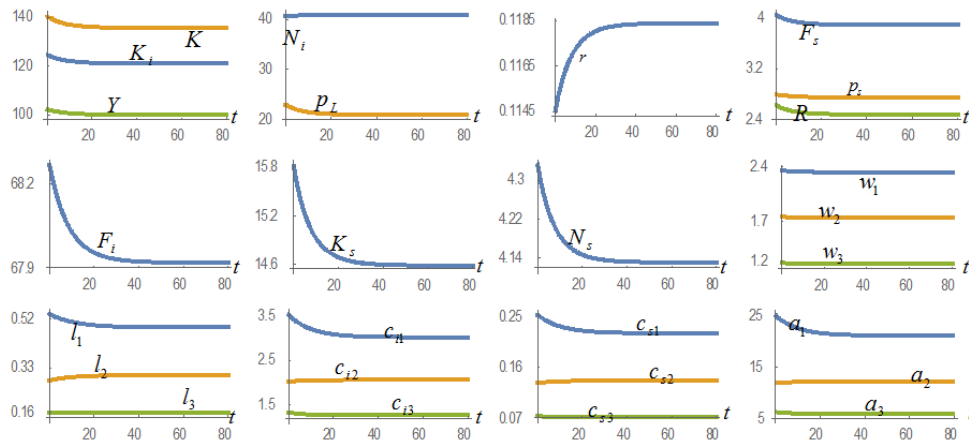The motion of the economic system is plotted in Figure 1.

Figure no. 1 – The Motion of the Economic System

In Figure 1, $Y(t)$ stands for the national product, defined as

$$Y(t) \equiv F_i(t) + p_s(t)F_s(t) + R(t)\sum_{j=1}^{J} l_j(t)\overline{N}_j.$$

It should be noted that the dynamic relationship between the GDP and the land price plotted in Figure 1 has obtained much attention in economic literature. Liu *et al.* (2011: 1) observe: "The recent financial crisis caused by a collapse of the housing market propelled the U.S. economy into the Great Recession. A notable development during the crisis period was a slump in business investment in tandem with a sharp decline in land prices." The conclusions made by Liu *et al.* are based on the data for the Great Recession period as well as for the entire sample period from 1975 to 2010. The simulation confirms that the dynamic system achieves a stationary state by t=80. From Figure 1 we see that the initial values of the land rent and price of services is fixed higher than their long-term equilibrium values. The two

variables fall over time. The output levels of the two sectors fall. The national product falls over time. Group 1's and Group 3's wealth levels fall, while Group 2's wealth level rises slightly. Group 1's lot size is reduced, Group 2's lot size is increased, and Group 3's lot size is slightly affected. We confirm that the system achieves at a stationary state in the long term. Simulation finds the following equilibrium values of the variables

$$r = 0.118, \ p_s = 2.75, \ p_L = 20.93, \ R = 2.48, \ Y = 99.96, \ K = 135.58, \ F_i = 67.92, \ F_s = 3.89,$$
$$N_i = 40.87, \ N_s = 4.13, \ L_s = 1.38, \ K_i = 121, \ K_s = 14.59, \ w_1 = 2.33, \ w_2 = 1.75, \ w_3 = 1.16,$$
$$a_1 = 21.12, \ a_2 = 12.19, \ a_3 = 5.87, \ l_1 = 0.49, \ l_2 = 0.3, \ l_3 = 0.16, \ c_{i1} = 3.02, \ c_{i2} = 2.06,$$
$$c_{i3} = 1.27, \ c_{s1} = 0.22, \ c_{s2} = 0.14, \ c_{s3} = 0.07. \tag{21}$$

We also calculate the three eigenvalues as follows

$$\{-0.16, -0.12, -0.10\}.$$

Hence, the equilibrium point is stable. The existence of a unique stable equilibrium point is important as we can effectively conduct comparative dynamic analysis.

4. COMPARATIVE DYNAMIC ANALYSIS

We plotted the motion of the economic system in the previous section. This section conducts comparative dynamic analysis, demonstrating how a change in a parameter alternates paths of the economic growth. As we can describe the motion of the system for any set of parameters, it is straightforward to make comparative dynamic analysis. This study uses the variable, $\overline{\Delta}x(t)$, to represent the change rate of the variable, $x(t)$, in percentage due to changes in the parameter value.

A rise in Group 1's population

The impact of population growth on economic structure and growth is a challenging question in theoretical economics. Although many empirical studies show that the effect of population growth may be either positive or negative and can be positive. Theoretical models with human capital predict situation-dependent interactions between population and economic growth (see, Ehrlich and Lui, 1997; Galor and Weil, 1999; Boucekkine, *et al.*, 2002; Bretschger, 2013). There are also mixed conclusions in empirical studies on the issue (e.g., Furuoka, 2009; Yao *et al.*, 2013). Our model allows us to examine how each group's population may affect growth and inequality. As our model is built with heterogeneous households, we can effectively study effects of changes in any group's population. We now examine what will happen to the motion of the economic variables if Group $1's$ population is changed as follows: $N_1 : 5 \Rightarrow 5.2,$ where "$\Rightarrow$" stands for "being changed to". The impacts on the variables are plotted in Figure 2. The change in the population has no impact on the land distribution between service use and residential use. As far as macroeconomic variables are concerned, the population growth has positive effects. The physical wealth and GDP are increased. The levels of output and all the input factors of the two sectors are increased. The rate of interest falls. The land value, land rent and price of services are all enhanced. The wage rates

are increased in association with capital increase. Nevertheless, the population growth has negative effects on the microeconomic variables. The consumption levels of goods and services, lot sizes, and wealth levels of each group are reduced.
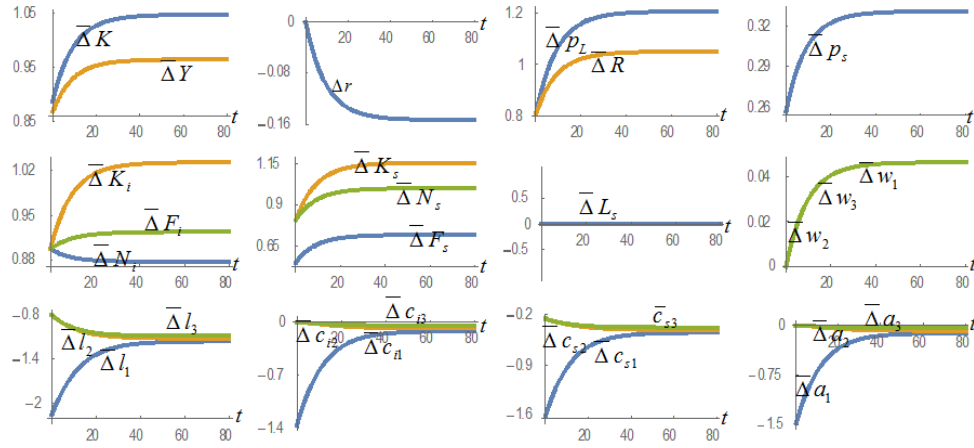

Figure no. 2 – A Rise in Group 1's Population

Group 3 augmenting human capital

Relations between human capital and economic growth are significant issues in modern economic theory and empirical research. It is widely believed in economics that human capital is an important factor for economic development (e.g., Hanushek and Kimko, 2000; Barro, 2001; Krueger and Lindahl, 2001; Castelló-Climent and Hidalgo-Cabrillana, 2012). As observed by Tilak (1989) inequality is negatively related to spread education within countries. Could *et al.* (2001) find that the primary source of inequality growth within uneducated workers is due to increasing randomness, but inequality growth within educated workers is mainly due to changes in the composition and return to ability (see also Tselios, 2008; Fleisher *et al.* 2011). To examine issues related to growth, inequality and human capital, we now allow Group 3 to improve its human capital as follows: $h_3 : 1 \Rightarrow 1.1$. The impacts on the variables are plotted in Figure 3. Group $3's$ wage rate is increased, while the other two groups' wage rates are slightly affected. The change in the population has almost no impact on the land distribution between service use and residential use. As far as macroeconomic variables are concerned, the enhancement in human capital has positive effects. The physical wealth and GDP are increased. The levels of output and all the input factors of the two sectors are increased. The rate of interest falls slightly in initial stage and then rises. The land value, land rent and price of services are all enhanced. The effects on the macroeconomic variables are similar the effects of the change in the population discussed before. Nevertheless, the effects on the microeconomic variables are obviously different. The consumption levels of goods and services, lot size, and wealth level of group 3 are augmented. The other two groups' lot sizes and consumption of services are reduced in the long term. The other two groups' consumption of goods and wealth levels are slightly affected in the long term. Hence, in the long term a rise in Group 3 benefits the group but harms the other two groups in the long term.
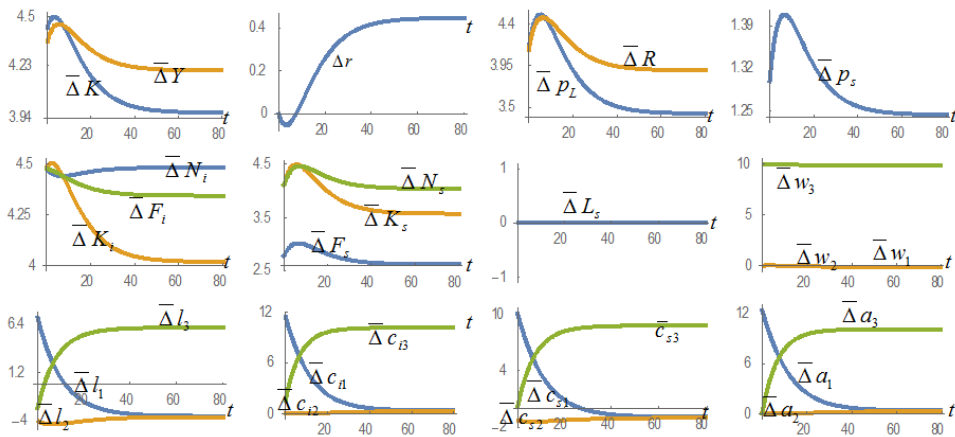
Figure no. 3 – A Rise in Group 3's Human Capital

A rise in the service output elasticity of land

To examine how land value is determined, we now allow the output elasticity of land to rise as follows: $\gamma_s : 0.32 \Rightarrow 0.33$. To keep constant return to scale, we also specify: $\alpha_s : 0.23 \Rightarrow 0.22$. The impacts on the variables are plotted in Figure 4. The rise in the output elasticity of land shifts the land distribution more to service production. The labor distribution is slightly affected. The industrial sector is slightly affected. The fall in the output elasticity of capital in the service sector reduces the sector's capital input. The sector's output is also reduced. The GDP is slightly affected. The national physical wealth is reduced in association with rising rate of interest. The price of services is increased. The land rent and value are enhanced. The wage rates are reduced slightly. The lot sizes of and consumption levels of services by the three groups are all reduced. The wealth and consumption levels of industrial goods are increased for all the three groups.
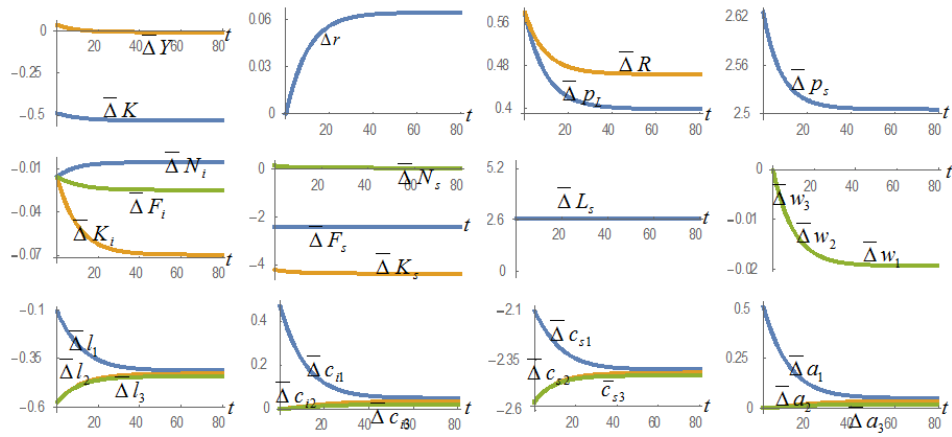

Figure no. 4 A Rise in the Service Output Elasticity of Land

A rise in Group 1's propensity to consume services

Different preferences of different households are important for analyzing economic equilibrium and structure in the Walrasian general equilibrium theory. Nevertheless, the Walrasian theory does not contain proper economic mechanisms for analyzing effects of changes in one type of households on national economic growth as well as wealth and income distribution among different households. As our analytical framework integrates the economic mechanism of the Walrasian general equilibrium theory and neoclassical growth theory, in principle we can analyze effects a change in the preference of any people on the dynamic path of the economic growth. We now allow Group 1's propensity to consumer services as follows: $\gamma_{01} = 0.02 \Rightarrow 0.025$. The rise in the propensity to consume services initially increases but soon reduces the national physical wealth and GDP. The rate of interest is reduced. The land rent and value are increased initially and reduced in the long term. The price of services is increased initially and reduced in the long term. The output and two inputs of the service sector are augmented. The output and two inputs of the industrial sector are lowered. The representative household from Group 1 consumes more services. The household initially lives in a larger house, consumes more industrial goods, and owns more wealth, in the long term lives in a smaller house, consumes less industrial goods, and owns less wealth. The household's wealth change occurs as the land value and rent are increased largely in initial stage. In the long term the land rent is slightly affected but the land value is largely reduced. The price of services is slightly affected. This implies that the household consumes more industrial goods as the wealth is increased and the wage rate is slightly affected. The other two groups whose preferences are not changed have less wealth, consume larger lot sizes and more industrial goods in the long term.
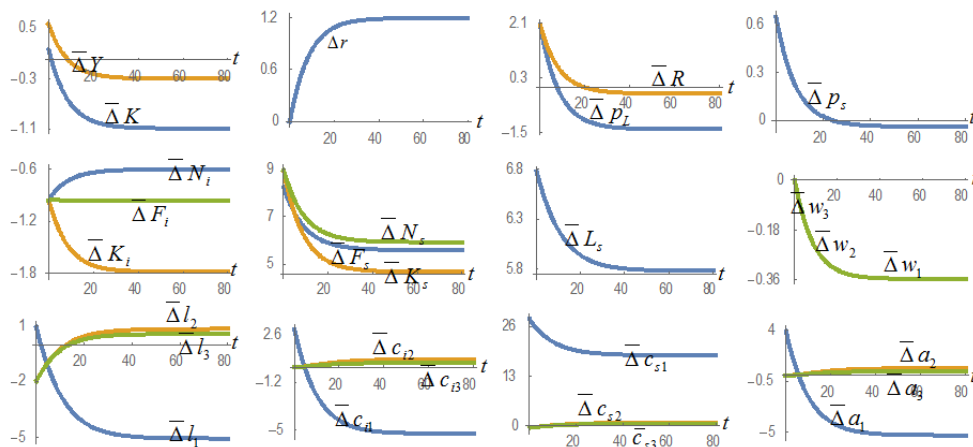

Figure no. 5 – A rise in Group 1's propensity to consume services

A rise in Group 1's propensity to consume housing

We now allow Group 1's propensity to consume housing as follows: $\eta_{01} = 0.04 \Rightarrow 0.05$.

The change in the preference augments the land rent and the price of services and lowers the wage rates. The land value is increased initially and is almost not affected in the long term. The GDP is enhanced initially and almost affected in the long term. The physical wealth is reduced. More land is for housing and less for agricultural production. The labor force is shifted from

industrial sector to the service sector initially. The land distribution is not affected in the long term. The output levels of the two sectors are reduced in the long term. The household from Group 1 lives in larger houses and the other two groups in smaller houses.
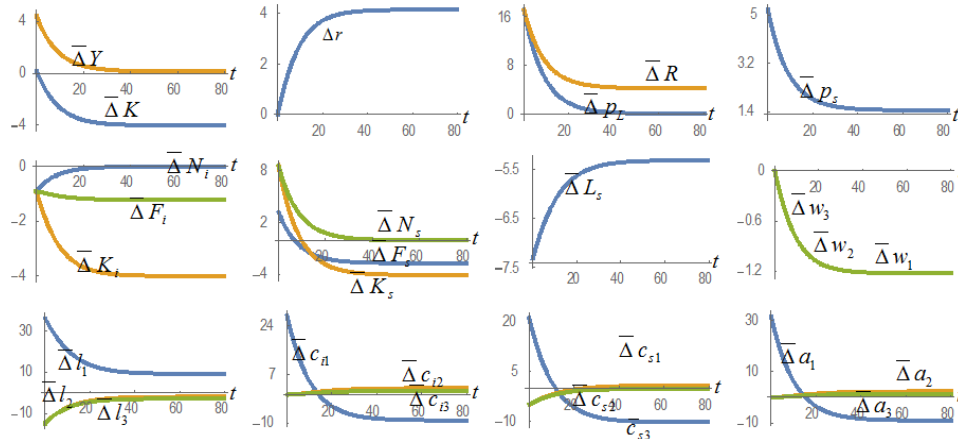

Figure no. 6 – A Rise in Group 1's Propensity to Consume Housing

Group 1 augmenting the propensity to save

We now examine the case that Group 1 increases its propensity to save in the following way: $\lambda_{01}: 0.7 \Rightarrow 0.71$. The simulation results are given in Figure 7. Group 1 increases the propensity to save, the household from the group initially experience lower consumption levels of lot size, services and goods. But as the household accumulates more wealth, the household consumes more of these goods. In the long term the other two groups lose due to Group 1's preference change as they have less wealth, live in smaller houses, consumer less goods and services. The physical wealth and GDP are enhanced in the long term. The rate of interest falls and the wage rates are increased. The land rent and land value fall initially and are enhanced in the long term. The output and capital inputs of the two sectors are augmented. In the long term the other two groups suffer from Group 1's preference change as their lot sizes, consumption levels of services and goods, and wealth are all reduced.
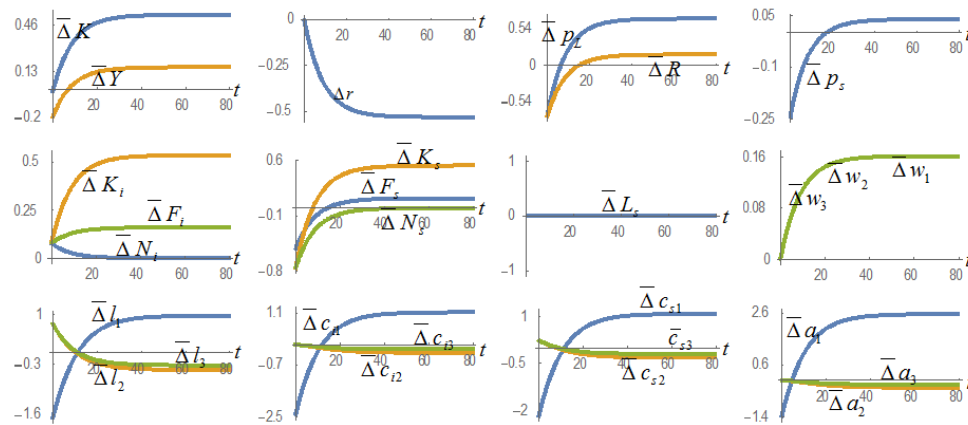

Figure no. 7 – Group 1 Augmenting the Propensity to Save

5. CONCLUSIONS

This paper proposed a dynamic model to examine the relationship between growth and inequality in a two-sector growth modelling framework. A new aspect of the study is to determine land value and rent and how these variables are related to growth and inequality. The production side consists of one service sector and one industrial sector. Land supply is fixed and nondepreciating. It is distributed between service production and residential use. The basic economic structure is based on the Walrasian general equilibrium theory and wealth accumulation is based the Solow-Uzawa neoclassical growth theory. We used an alternative utility function proposed by Zhang to describe the behavior of households. In our approach the wealth and income inequality is due to heterogeneity in households' preferences and human capital levels as well as the households' initial wealth. We first built a model for any number of types of household with endogenous wealth and land distribution. Then we gave a computation procedure for simulating model with proper initial conditions. For illustration we simulated the model for the economy with three types of household. The system has a unique stable equilibrium point for the given parameters. We simulated the motion of the national economy and carried out comparative dynamic analysis with regard to changes. The comparative dynamic analysis provides some important insights. For instance, as the rich group increases its propensity to save, the household from the group initially experiences lower consumption levels of lot size, services and goods. But as the household accumulates more wealth, the household consumes more lot size, goods and services. In the long term the other two groups suffer from the rich people's preference change as their lot sizes, consumption levels of services and goods, and wealth are all reduced. The physical wealth and GDP are enhanced in the long term. The rate of interest falls and the wage rates are increased. The land rent and land value fall initially and are enhanced in the long term. The output and capital inputs of the two sectors are augmented. By the way, our results on relations between economic growth and land value (which are reflected in Figures 2-7) are similar to the phenomenon that is described by Liu *et al.* (2011: 1), "The recent financial crisis caused by a collapse of the housing market propelled the U.S. economy into the Great Recession. A notable development during the crisis period was a slump in business investment in tandem with a sharp decline in land prices." The similarity is concluded in the sense that a rising period in the GDP is in tandem with an increasing period in the land price. It should be also mentioned that issues related to determination land values (and other assets such as gold) are addressed in different studies (e.g., Feldstein, 1980). But almost all of these studies are developed in analytical frameworks without endogenous wealth accumulation in a general dynamic equilibrium model. Although this study analyzes land value and rent in a general dynamic economic model, the following directions for further research raised by Feldstein (1980: 317) remain to be done in the future: "It would clearly be desirable to extend the current model by developing an explicit theory of portfolio equilibrium for investors who hold land, gold, bonds, and equity shares. The real yields on these assets would be linked because they are all dependent upon future changes in expected inflation. As a further step, the analysis should recognize that the effect of inflation on each individual's demand for each asset depends on that individual's own tax situation." It is a great challenge to develop a far more general economic theory which includes a "theory of portfolio equilibrium for investors who hold land, gold, bonds, and equity shares" in a general dynamic equilibrium theory which treats value determinants of depreciating and non depreciating assets within a consistent framework with endogenous accumulation of wealth, environment and different resources.

Appendix: Proving the Lemma

We now confirm the lemma in section 3. From (3) and (5), we obtain

$$z \equiv \frac{r + \delta_k}{w} = \frac{\tilde{\alpha}_i N_i}{K_i} = \frac{\tilde{\alpha}_s N_s}{K_s}, \tag{A1}$$

where we omit time index and $\tilde{\alpha}_j \equiv \alpha_j / \beta_j$, $j = i$, s. By (2) and (3), we have

$$r + \delta_k = \frac{\alpha_i A_i}{\tilde{\alpha}_i^{\beta_i}} z^{\beta_i}, \quad w = \frac{\tilde{\alpha}_i^{\alpha_i} \beta_i A_i}{z^{\alpha_i}}, \tag{A2}$$

where we also use (A1). We express w and r as functions of z.

From (11) and (13), we get

$$\sum_{j=1}^{J} \gamma_j \hat{y}_j \overline{N}_j = p_s F_s. \tag{A3}$$

From (4), we have

$$r + \delta_k = \frac{\alpha_s p_s F_s}{K_s}. \tag{A4}$$

From (A4) and (5) we solve

$$p_s \left(\frac{L_s}{K_s} \right)^{\gamma_s} = \frac{\tilde{\alpha}_s^{\beta_s} (r + \delta_k)}{\alpha_s A_s z^{\beta_s}}, \tag{A5}$$

where we use (A1). Insert (A1) in $N_i + N_s = N$

$$\left(\frac{K_i}{\tilde{\alpha}_i} + \frac{K_s}{\tilde{\alpha}_s} \right) z = N. \tag{A6}$$

Solve $K_i + K_s = K$ and (A6) with K_i and K_s as the variables

$$K_i = \frac{\tilde{\alpha} N}{z} - \frac{\tilde{\alpha} K}{\tilde{\alpha}_s}, \quad K_s = \frac{\tilde{\alpha} K}{\tilde{\alpha}_i} - \frac{\tilde{\alpha} N}{z}, \tag{A7}$$

where we use (15) and

$$\tilde{\alpha} \equiv \left(\frac{1}{\tilde{\alpha}_i} - \frac{1}{\tilde{\alpha}_s} \right)^{-1}.$$

By (A8), we solve the capital distribution as functions of z and K. By (A1), we solve the labor distribution N_i and N_s as functions of z and K as follows

$$N_i = \left(N - \frac{zK}{\tilde{\alpha}_s} \right) \frac{\tilde{\alpha}}{\tilde{\alpha}_i}, \quad N_s = \left(\frac{zK}{\tilde{\alpha}_i} - N \right) \frac{\tilde{\alpha}}{\tilde{\alpha}_s}. \tag{A8}$$

By (5) and (A3), we have

$$L_s = \frac{\gamma_s}{R} \sum_{j=1}^{J} \gamma_j \, \hat{y}_j \, \overline{N}_j. \tag{A9}$$

From $l_j R = \eta_j \, \hat{y}_j$ in (11) and (A9), we have

$$L_s = \gamma_s \sum_{j=1}^{J} \frac{\gamma_j \, \overline{N}_j \, l_j}{\eta_j}. \tag{A10}$$

From (A10) and (18)

$$\sum_{j=1}^{J} \overline{\gamma}_j \, l_j = L, \tag{A11}$$

where

$$\overline{\gamma}_j \equiv \left(1 + \frac{\gamma_j \gamma_s}{\eta_j} \right) \overline{N}_j.$$

From (8) and (9)

$$\hat{y}_j = r \overline{k}_j + R \overline{l}_j + h_j w + a_j. \tag{A12}$$

Insert (6) in (A12)

$$\hat{y}_j = (1 + r)a_j + h_j w. \tag{A13}$$

From $l_j R = \eta_j \hat{y}_j$ in (11) and (A13)

$$l_j R = (1 + r)\eta_j a_j + \eta_j h_j w. \tag{A14}$$

Multiply (A14) by $\bar{\gamma}_j$

$$\bar{\gamma}_j l_j = \frac{(1 + r)\eta_j \bar{\gamma}_j a_j + \eta_j \bar{\gamma}_j h_j w}{R}. \tag{A15}$$

Adding all the equations in (A15) and using (A11), we have

$$R = (1 + r)\sum_{j=1}^{J} \bar{\eta}_j a_j + \bar{\eta} w, \tag{A16}$$

where

$$\bar{\eta}_j \equiv \frac{\bar{\gamma}_j \eta_j}{L}, \quad \bar{\eta} \equiv \sum_{j=1}^{J} \bar{\eta}_j h_j.$$

Multiply (14) by p_L

$$\sum_{j=1}^{J} p_L \bar{l}_j \bar{N}_j = p_L L. \tag{A17}$$

Add (A17) and (16)

$$\sum_{j=1}^{J} a_j \bar{N}_j = K + \frac{RL}{r}. \tag{A18}$$

where we also use (6). Insert (A16) in (A18)

$$K = \sum_{j=1}^{J} a_j R_j - \frac{\bar{\eta} wL}{r}, \tag{A19}$$

where

$$R_j \equiv \bar{N}_j - \left(\frac{1 + r}{r}\right)\bar{\eta}_j L.$$

From (5) we have

$$w N_s = \frac{\beta_s L_s R}{\gamma_s}. \tag{A20}$$

Insert (A9) and (A8) in (A20)

$$K - \frac{\tilde{\alpha}_i N}{z} = \frac{\gamma_0}{z} \sum_{j=1}^{J} \frac{\gamma_j \hat{y}_j \overline{N}_j}{w}, \tag{A21}$$

where

$$\gamma_0 \equiv \frac{\tilde{\alpha}_i \tilde{\alpha}_s \beta_s}{\tilde{\alpha}}.$$

Insert (A19) in (A21)

$$\frac{\tilde{\alpha}_i N}{z} + \frac{\gamma_0}{z} \sum_{j=1}^{J} \frac{\gamma_j \hat{y}_j \overline{N}_j}{w} = \sum_{j=1}^{J} a_j R_j - \frac{\overline{\eta} w L}{r}, \tag{A22}$$

From (A22) and (A13) we solve

$$a_1 = \varphi\big(z, \{a_j\}\big) \equiv \left(\frac{\tilde{\alpha}_i N}{z} + \frac{h_0}{z} + \frac{\overline{\eta} w L}{r} - \sum_{j=2}^{J} \overline{R}_j a_j \right) \frac{1}{\overline{R}_1}, \tag{A23}$$

where

$$\overline{R}_j \equiv R_j - \left(\frac{1+r}{w} \right) \frac{\gamma_0 \gamma_j \overline{N}_j}{z}, \quad h_0 \equiv \gamma_0 \sum_{j=1}^{J} h_j \gamma_j \overline{N}_j.$$

It is straightforward to check that all the variables can be expressed as functions of z and $\{a_j\}$ at any point in time as follows: r and w by (A2) $\to a_1$ by (A23) $\to \hat{y}_j$ by (A12) $\to R$ by (A16) $\to K$ by (A19) $\to K_i$ and K_s by (A7) $\to N_i$ and N_s by (A1) $\to p_L$ by (6) $\to L_s$ by (A9) $\to p_s$ by (A5) $\to F_i$ by (2) $\to F_s$ by (4) $\to c_{sj}$, c_{ij}, l_j and s_j by (11).

From this procedure and (12)

$$\dot{a}_1 = \Omega_0\big(z, \{a_j\}\big) \equiv \lambda_1 \hat{y}_1 - a_1, \tag{A24}$$

$$\dot{a}_j = \Omega_j\left(z, \{a_j\}\right) \equiv \lambda_j\, \hat{y}_j - a_j, \quad j = 2, ..., J. \tag{A25}$$

Taking derivatives of (A21) with respect to time yields

$$\dot{a}_1 = \frac{\partial \varphi}{\partial z} \dot{z} + \sum_{j=2}^{J} \Omega_j \frac{\partial \varphi}{\partial a_j}, \tag{A26}$$

where we use (A23). Equal (A22) and (A24)

$$\dot{z} = \Omega_1\left(z, \{\bar{k}_j\}\right) \equiv \left(\Omega_0 - \sum_{j=2}^{J} \Omega_j \frac{\partial \varphi}{\partial a_j}\right)\left(\frac{\partial \varphi}{\partial z}\right)^{-1}. \tag{A27}$$

We thus proved the lemma.

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