Scientific Annals
of the "Alexandru Ioan Cuza" University of Iaşi
Economic Sciences
62 (3), 2015, 391-410
DOI 10.1515/aicue-2015-0027

A SYNTHESIS OF THE HECKSCHER-OHLIN AND ONIKI-UZAWA TRADE MODELS WITH HETEROGENEOUS TASTES, DIFFERENT TECHNOLOGIES, AND ENDOGENOUS WEALTH

Wei-Bin ZHANG*

Abstract

This paper examines the role of preferences and technological differences between two countries in determining dynamics of global wealth and pattern of trade in a reformed H-O model of international trade. The paper builds a trade model with endogenous wealth accumulation and labor and capital distribution between sectors and between countries under perfectly competitive markets and free trade. The model is based the H-O model, the Solow-Uzawa neoclassical growth model and the Oniki-Uzawa trade model. Each country has three sectors, producing one globally homogenous tradable capital good, specifying in producing one-tradable commodity, and supplying non-tradable goods and services. The study simulates the model for the economy to demonstrate existence of equilibrium points and motion of the dynamic system. It examines effects of changes in output elasticity of an industrial sector, population expansion, and propensities to consume the domestic commodity, to consume the other country's commodity, to consume services, and to hold wealth.

Keywords: trade pattern; O-H model; Oniki-Uzawa model; economic growth; wealth accumulation

JEL classification: F11, F21

1. INTRODUCTION

The Heckscher-Ohlin (H-O) model is one of the core models in model trade theories. A standard H-O model is for a two-countries global economy, each country having access to the same technology for producing two goods using two fixed factors (labor and capital) under conditions of perfect competition and constant returns to scale. Factors of production are mobile between sectors within a country, but immobile internationally. No international borrowing and lending are allowed. As pointed out by Ethier (1974), this theory has four "core proportions". In the simple case of two-commodity and two-country world economy, these four propositions are as follows: (1) factor-price equalization theorem by Lerner (1952) and Samuelson (1948, 1949), implying that free trade in final goods alone brings about complete international equalization of factor prices; (2) Stolper-Samuelson theory by Stolper and Samuelson (1941), predicting that an increase in the relative price of one

* Ritsumeikan Asia Pacific University, Japan; e-mail: *wbz1@apu.ac.jp*.

commodity raises the real return of the factor used intensively in producing that commodity and lowers the real return of the other factor; (3) Rybczynski theorem by Rybczynski (1955), stating that if commodity prices are held fixed, an increase in the endowment of one factor causes a more than proportionate increase in the output of the commodity which uses that factor relatively intensively and an absolute decline in the output of the other commodity; and (4) Heckscher-Ohlin theorem by Heckscher (1919) and Ohlin (1933; see also Heckscher and Ohlin, 1991), proving that a country tends to have a bias towards producing and exporting the commodity which uses intensively the factor with which it is relatively well-endowed. The model explains patterns of trade based on the factor endowments of countries. According to Chen (1992, pp. 923-924), "It appears to be the general consensus in this body of literature that the main determinant of long-run comparative advantage is the countries' savings rates. The question of what has caused the difference in savings rates among countries, however, is rarely explicitly discussed in the literature. The models that do endogenize savings rates (e.g., Stiglitz, 1970) attribute the difference in savings rates and hence long-run comparative advantage to a difference in preferences; in particular, a difference in agents' time discount factors among countries. Yet explaining trade in terms of differences in preferences is no longer in the spirit of the Heckscher-Ohlin model in which trade arises because of differences in relative factor proportions." There is a large literature related to introducing dynamic elements to H-O models. An early example is given by Chen (1992) whose purpose is study long-run equilibria in open economies in an H-O modelling framework with endogenous savings and endogenous labor supply. The model provides an H-O type explanation for long-run trade between countries with identical preference. Baxter (1992) builds a model structurally similar to Chen's with tax rates being different across countries. The pattern of trade and specification is determined by the taxes in steady states. Ventura (1997) builds a model by combining a weak form of the factor-price-equalization theorem of international trade with the Ramsey model of economic growth. Bajona and Kehoe (2010) develop a dynamic H-O by combining a static two-good, two-factor H-O trade model and a two-sector growth model in infinitely lived consumers where borrowing and lending are not permitted. There are other models which attempt to generalize or extend the H-O model in different directions (Zhang, 2008). These H-O models don't allow international factor mobility and don't allow borrowing and lending. Observing goods and capital flows between countries in contemporary world, we see evidently that capital immobility is not a realistic assumption. This study relaxes this assumption. In particular, we apply the utility function proposed by Zhang (1993) to develop a dynamic H-O model.

The model in this study is as Ricardian as it postulates cross-country differences in technology and labor productivity, without providing endogenous determinants of the differences. We consider endogenous wealth as a main engine of global economic growth. We describe international trade on the basis of the dynamic model with accumulating capital developed by Oniki and Uzawa and others (for instance, Oniki and Uzawa, 1965; Frenkel and Razin, 1987; Sorger, 2003; and Nishimura and Shimomura, 2002). The Oniki-Uzawa model is constructed for the two-country with two goods with fixing saving rates. Deardorff and Hanson (1978) construct a model of different fixed rates in which the country with the higher savings rate exports the capital intensive good in steady state. The Oniki-Uzawa model is often used as a start point for analyzing interdependence between trade patterns and economic growth. In most of this type models goods and services are classified into capital goods and consumer goods. Nevertheless, it has been recorded that a high share of GDP in modern economies is non-tradable. Distinction between tradable good and non-tradable good is significant for explaining the terms of trade (Mendoza, 1995), for explaining the exchange rate (Stulz, 1987; Stockman and Dellas, 1989; Backus and Smith, 1993; Rogoff, 2002); for dealing with current account dynamics (Edwards, 1989), or for solving the home premium puzzle (Baxter *et al*., 1998; Pesenti and Van Wincoop, 2002). Backus and Smith (1993) emphasized the significance of this distinction as follows: "The mechanism is fairly simple. "Although the law of one price holds, in the sense that each good sells for a single price in all countries, PPP may not: price indexes combine prices of both traded and nontraded goods, and because the latter are sold in only one country their prices, and hence price indexes, may differ across countries". There are also other studies emphasizing distinction between tradable and non-tradable sectors (Stockman and Tesar, 1995; Zhao *et al*., 2014). This study introduces distinct sectors to examine trade patterns and economic dynamics.

As the model in this study is a dynamic general equilibrium model, it can properly address issues related to income convergence between countries. Discussions about these issues are mostly based on the insights from analyzing models of closed economies in the literature of economic growth and development (Barro and Sala-i-Martin, 1995). One might reasonably expect that one can get little proper insights into the convergence issues with a framework without international interactions. This paper develops a two-country growth trade model with economic structure, treating the global economy as an integrated whole. We analyze trade issues within the framework of a simple international macroeconomic growth model with perfect capital mobility. The model in this study is a further development of the two models by Zhang. Zhang (2012, 2014) proposes a multi-country model with capital accumulation and knowledge with traditional one-tradable capital good and one non-tradable good framework. This paper develops an international growth model with three sectors in each national economy by adding one country-specified tradable good into the analytical framework. The introduction of this good makes the modelling framework more robust in exploring complexity of global economic growth. The rest of the paper is organized as follows. Section 2 defines the basic model. Section 3 shows how we solve the dynamics and simulates the motion of the global economy. Section 4 carries out comparative dynamic analysis to examine the impact of changes in some parameters on the motion of the global economy. Section 5 concludes the study. The appendix proves the main results in Section 3.

2. THE MODEL

Our model is influenced by some typical dynamic H-O models and the neoclassical trade growth theory. For simplicity, we consider a world economy with two national economies, indexed by $j=1, 2$. Rather than two goods in the standard H-O model, this study assumes that each national economy produces three goods. Both countries produce a homogeneous capital consumer goods which can be used as capital and consumer goods. The sectors in the two countries are called industrial sector. This sector is similar to the homogenous sector in the traditional neoclassical trade model (e.g., Ikeda and Ono, 1992). Capital goods are freely mobile between countries. There is no tariff on any good in the global economy. Each country provides services and country-specified goods which are not internationally tradable and can be consumed only by domestic households. Each country specifies in producing a good called global commodity which is internationally tradable and consumed by both countries. Global commodities are pure consumption goods. The industrial sector, global commodity sectors, and service sectors are indexed by i, j, s. Production sectors use capital and labor. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Like in the traditional H-O model, labor is internationally immobile. Capital and labor are freely mobile within each country and are immobile between countries. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to factors of production. We omit the possibility of hoarding of output in the form of non-productive inventories held by households. We describe capital sector and wealth accumulation as in the neoclassical growth theory. Most neoclassical growth models are based on the pioneering works of Solow (1956).

Let all the prices be measured in terms of capital good. We use $p_j(t)$ to represent the price of country $j's$ services. The price of global commodity supplied by country j is denoted by $p_{jj}(t)$. As labor is immobile between countries, wages vary between countries. Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. We denote the wage rate in the j th country and interest rate by $w_j(t)$ and $r(t)$, respectively. Capital depreciates at a constant exponential rate δ_{jk}, being independent of the manner of use within each country. Depreciation rates may vary between countries. We use $N_{jq}(t)$ and $K_{jq}(t)$ to stand for the labor force and capital stocks employed by sector q in country j. Let $F_{jq}(t)$ stand for the output level of sector q in country j.

Production functions

The production functions are neoclassical and homogeneous of degree one with the inputs. The production function of section q in country j is specified as

$$F_{jq}(t) = A_{jq}\, K_{jq}^{\alpha_{jq}}(t) N_{jq}^{\beta_{jq}}(t), \quad A_{jq},\, \alpha_{jq},\, \beta_{jq} > 0, \quad \alpha_{jq} + \beta_{jq} = 1 \tag{1}$$

where A_{jq}, α_{jq}, and β_{jq} are positive parameters. Different from the Richardian trade theory which assumes technological differences between countries, the H-O model assumes variations in capital and labor endowments with the identical technology between countries. In this study we assume not only differences in capital and labor endowments like in the H-O model but also differences in technologies between countries like in the Ricardian trade theory.

Marginal conditions for industrial sectors

The rate of interest, wage rate, and prices are determined by markets. Hence, for any individual firm rate of interest, wage rate, and prices are given at each point of time. The industrial sector chooses the two variables $K_{ji}(t)$ and $N_{ji}(t)$ to maximize its profit. The marginal conditions are

$$r(t) + \delta_{jk} = \frac{\alpha_{ji}\, F_{ji}(t)}{K_{ji}(t)}, \quad w_j(t) = \frac{\beta_{ji}\, F_{ji}(t)}{N_{ji}(t)}, \quad j = 1, 2. \tag{2}$$

Marginal conditions for the global commodity sectors

The marginal conditions for global commodity sector j in country j are

$$r(t) + \delta_{jk} = \frac{\alpha_{jj} \; p_{jj}(t) F_{jj}(t)}{K_{jj}(t)}, \quad w_j(t) = \frac{\beta_{jj} \; p_j(t) F_{jj}(t)}{N_{jj}(t)}, \quad j = 1, 2 \tag{3}$$

Marginal conditions for service sectors

The marginal conditions for service sectors are

$$r(t) + \delta_{jk} = \frac{\alpha_{js} \; p_{js}(t) F_{js}(t)}{K_{js}(t)}, \quad w_j(t) = \frac{\beta_{js} \; p_{js}(t) F_{js}(t)}{N_{js}(t)}, \quad j = 1, 2 \tag{4}$$

The current income and disposable income

This study uses Zhang's utility function to describe behavior of households (Zhang, 1993). Consumers make decisions on consumption levels of goods and saving. Let $\bar{k}_j(t)$ stand for the wealth of household in country j. Per household's current income from the interest payment $r(t)\bar{k}_j(t)$ and the wage payment $w_j(t)$ is

$$y_j(t) = r(t)\bar{k}_j(t) + w_j(t).$$

Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The per capita disposable income is given by

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t) = (1 + r(t))\bar{k}_j(t) + w_j(t). \tag{5}$$

The disposable income is used for saving and consumption.

The budgets and utility functions

Let $c_{jq}(t)$ stand for consumption level of consumer good q in country j. We use $s_j(t)$ to stand for the saving made at the current time by the representative household in country j. The consumer j is faced with the following budget constraint

$$c_{ji}(t) + p_1(t)c_{j1}(t) + p_2(t)c_{j2}(t) + p_{js}(t)c_{js}(t) + s_j(t) = \hat{y}_j(t). \tag{6}$$

We assume that consumers' utility function is a function of the consumption levels of goods、services and the saving as follows

$$U_j(t) = c_{ji}^{\xi_{ji0}}(t) c_{j1}^{\xi_{j10}}(t) c_{j2}^{\xi_{j20}}(t) c_{js}^{\gamma_{js0}}(t) s_j^{\lambda_{j0}}(t), \quad \xi_{ji0}, \xi_{j10}, \xi_{j20}, \xi_{js0}, \lambda_{j0} > 0, \tag{7}$$

where ξ_{ji0} is called the propensity to consume industrial goods, good 1, ξ_{jj0} the propensity to consume global commodity j, and λ_{j0} the propensity to own wealth. Maximizing (7) subject to (6) yields

$$c_{ji}(t) = \xi_{ji}\,\hat{y}_j(t), \quad p_1(t)c_{j1}(t) = \xi_{j1}\,\hat{y}_j(t), \quad p_2(t)c_{j2}(t) = \xi_{j2}\,\hat{y}_j(t), \quad p_{js}(t)c_{js}(t) = \xi_{js}\,\hat{y}_j(t),$$
$$s_j(t) = \lambda_j\,\hat{y}_j(t), \tag{8}$$

where

$$\xi_{ji} \equiv \rho_j\,\xi_{ji0}, \quad \xi_{j1} \equiv \rho_j\,\xi_{j10}, \quad \xi_{j2} \equiv \rho_j\,\xi_{j20}, \quad \xi_{js} \equiv \rho_j\,\xi_{js0}, \quad \lambda_j \equiv \rho_j\,\lambda_{j0},$$
$$\rho_j \equiv \frac{1}{\xi_{ji0} + \xi_{j10} + \xi_{j20} + \xi_{js0} + \lambda_{j0}}.$$

Wealth accumulation

According to the definitions of $s_j(t)$, the wealth change of the representative household in country j is

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \tag{9}$$

This equation simply states that the change in wealth is equal to saving minus dissaving.

Factor marketing clearing conditions

We use $K_j(t)$ to stand for the capital stocks employed by country j. The capital stock is fully employed by the three sectors. That is

$$K_{ji}(t) + K_{jj}(t) + K_{js}(t) = K_j(t), \quad j = 1, 2. \tag{10}$$

The labor force is fully employed by the three sectors

$$N_{ji}(t) + N_{jj}(t) + N_{js}(t) = N_j, \quad j = 1, 2. \tag{11}$$

Market clearing for two tradable goods

The demand and supply of services balance in each national market

$$c_{js}(t)N_j = F_{js}(t), \quad j = 1, 2. \tag{12}$$

Market clearing in global commodity markets

The demand and supply of tradable goods balance in global markets

$$c_{1q}(t)N_1 + c_{2q}(t)N_2 = F_{qq}(t), \quad q = 1, 2. \tag{13}$$

Market clearing in capital markets

The global capital production is equal to the global net savings. That is

$$\sum_{j=1}^{2}\left(s_j(t)N_j - \bar{k}_j(t)N_j + \delta_{kj}K_j(t)\right) = \sum_{j=1}^{2} F_{ji}(t). \tag{14}$$

Wealth balance

The wealth owned by the global population is equal to the total global wealth

$$\bar{k}_1(t)N_1 + \bar{k}_2(t)N_2 = K_1(t) + K_2(t) = K(t), \quad j = 1, 2. \tag{15}$$

We thus built the dynamic model with endogenous wealth accumulation.

3. THE DYNAMICS AND EQUILIBRIUM

We have the dynamic equations for the two-country global economy. As the system is nonlinear and is of high dimension, it is difficult to generally analyze behavior of the system. Before examining the dynamic properties of the system, we show that dynamics of the two-country economies can be expressed by 2 differential equations. Before further stating analytical results, we introduce a variable

$$z_1 \equiv \frac{r + \delta_{1k}}{w_1}.$$

The following lemma shows how to follow the dynamics of global economic growth with initial conditions.

Lemma 1

The motion of the 2 variables $z_1(t)$ and $\bar{k}_2(t)$ is given by the following 2 differential equations

$$\begin{aligned} \dot{z}_1(t) &= \tilde{\Lambda}_1\left(z_1(t), \bar{k}_2(t)\right), \\ \dot{\bar{k}}_2(t) &= \tilde{\Lambda}_2\left(z_1(t), \bar{k}_2(t)\right), \end{aligned} \tag{16}$$

where $\tilde{\Lambda}_1(t)$ and $\tilde{\Lambda}_2(t)$ are functions of $z_1(t)$ and $\bar{k}_2(t)$, defined in the appendix. The values of the other variables are given as functions of $z_1(t)$ and $\bar{k}_2(t)$ at any point in time by the following procedure: $r(t)$ by (A2) $\rightarrow$ $w_j(t)$ by (A2) $\rightarrow$ $p_j(t)$ by (A4) $\rightarrow$ $p_{js}(t)$ by (A5) $\rightarrow$ $\bar{k}_1(t)$ by (A7) $\rightarrow$ $K(t)$ by (A14) $\rightarrow$ $N_{jj}(t)$ by (A12) $\rightarrow$ $N_{ji}(t)$ by (A8) $\rightarrow$ N_{js} by (A7) $\rightarrow$ $K_{ji}(t)$, $K_{jj}(t)$, $K_{js}(t)$ by (A1)$\rightarrow$ $F_{jq}(t)$ by (1) $\rightarrow$ $\hat{y}_j(t)$ by (A5) $\rightarrow$ $K_j(t)$ by (10) $\rightarrow$ $c_{j1}(t)$, $c_{j2}(t)$ and $s_j(t)$ by (8).

For simulation, we specify values of the parameters as follows

$$N_1 = 10,\ N_2 = 20,\ \delta_{jk} = 0.05,\ A_{1i} = 1.2,\ A_{2i} = 1.3,\ A_{11} = 1.3,\ A_{22} = 1.2,\ A_{1s} = 1,$$
$$A_{2s} = 1,\ \alpha_{1i} = 0.32,\ \alpha_{2i} = 0.3,\ \alpha_{11} = 0.31,\ \alpha_{22} = 0.31,\ \alpha_{1s} = 0.29,\ \alpha_{2s} = 0.3,$$
$$\xi_{1i0} = 0.04,\ \xi_{110} = 0.05,\ \xi_{120} = 0.03,\ \xi_{1s0} = 0.03,\ \lambda_{1s0} = 0.75,\ \xi_{2i0} = 0.05,\ \xi_{210} = 0.03,$$
$$\xi_{220} = 0.05,\ \xi_{2s0} = 0.04,\ \lambda_{2s0} = 0.7. \tag{17}$$

Country 1 and $2's$ populations are respectively 10 and 20. We consider equal depreciation rates of physical capital between the countries and between sectors. The total factor productivities are different between the two economies. The total factor productivity of country 1's industrial sector is higher than that of Country 2's. The propensity to save of country 1's representative household is higher than that of Country 2's. We specify the initial conditions as follows:

$$z_1(0) = 0.05,\ \bar{k}_2(0) = 8.$$

The motion of the system is given in Figure 1. In the figure the national incomes and the global income are defined as follows:

$$Y_j \equiv F_{ji} + p_{js} F_{js} + p_j F_{jj},\ Y \equiv Y_1 + Y_2.$$

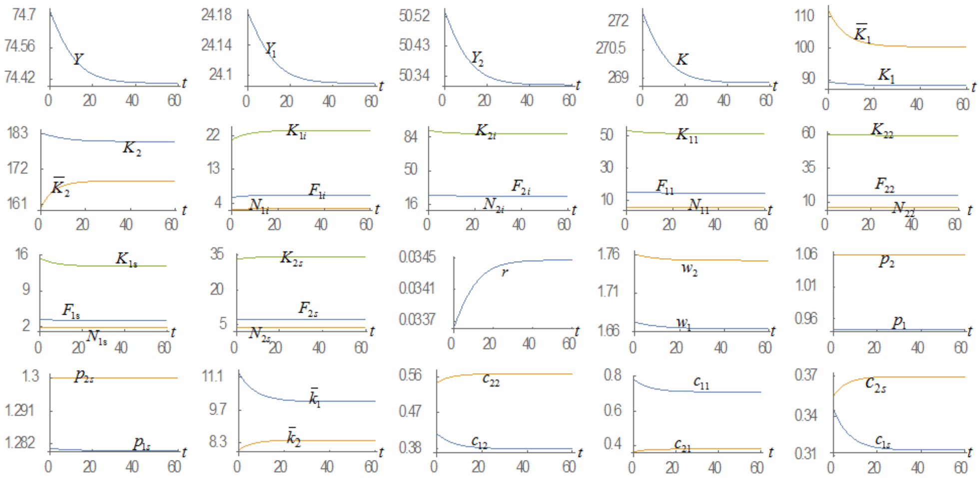

Figure no. 1 – The Motion of the Global Economy

The global income and wealth fall in association with rising rate of interest and falling wage rates. The national and global incomes fall over time. Country $1's$ wealth is more the capital stock employed by the national economy, implying that the country is in trade surplus. Country $2's$ wealth is less the capital stock employed by the national economy, implying that the country is in trade deficit. Country $1's$ wealth and the capital employed fall over time.

Country $2's$ wealth rises and the capital employed falls. Each sector also experiences changes over time as illustrated in Figure 1. The prices of two commodities and services are slightly changed. The consumption levels of commodities and services and wealth of country $1's$ ($2's$) representative household fall (rise) over time.

From Figure 1 we observe that the system becomes stationary in the long term. Following the procedure in the lemma, we calculate the equilibrium values of the variables as follows

$$Y = 74.4,\ Y_1 = 241,\ Y_2 = 50.3,\ K = 268.77,\ K_1 = 88.18,\ \bar{K}_1 = 100.53,\ K_2 = 180.59,$$
$$\bar{K}_2 = 168.24,\ r = 0.035,\ w_1 = 1.66,\ w_2 = 1.75,\ p_1 = 0.94,\ p_2 = 1.06,\ p_{1s} = 1.28,$$
$$p_{2s} = 1.3,\ c_{1i} = 0.54,\ c_{2i} = 0.6,\ c_{11} = 0.71,\ c_{21} = 0.38,\ c_{12} = 0.38,\ c_{22} = 0.57,$$
$$c_{1s} = 0.31,\ c_{2s} = 0.37,\ \bar{k}_1 = 10.05,\ \bar{k}_2 = 8.41,$$

$$\begin{pmatrix} F_{1i} \\ N_{1i} \\ K_{1i} \end{pmatrix} = \begin{pmatrix} 6.16 \\ 2.52 \\ 23.32 \end{pmatrix},\ \begin{pmatrix} F_{2i} \\ N_{2i} \\ K_{2i} \end{pmatrix} = \begin{pmatrix} 24.66 \\ 9.85 \\ 87.58 \end{pmatrix},\ \begin{pmatrix} F_{11} \\ N_{11} \\ K_{11} \end{pmatrix} = \begin{pmatrix} 14.74 \\ 5.77 \\ 51.06 \end{pmatrix},\ \begin{pmatrix} F_{22} \\ N_{22} \\ K_{22} \end{pmatrix} = \begin{pmatrix} 15.14 \\ 6.13 \\ 59.86 \end{pmatrix},\ \begin{pmatrix} F_{1s} \\ N_{1s} \\ K_{1s} \end{pmatrix} = \begin{pmatrix} 3.14 \\ 17.2 \\ 13.81 \end{pmatrix},\begin{pmatrix} F_{2s} \\ N_{2s} \\ K_{2s} \end{pmatrix} = \begin{pmatrix} 7.4 \\ 3.84 \\ 34.14 \end{pmatrix}.$$

It is straightforward to calculate the two eigenvalues as follows

$$\{-0.17,\ -0.14\}.$$

This implies that the world economy is stable. This implies that we can effectively conduct comparative dynamic analysis.

4. COMPARATIVE DYNAMIC ANALYSIS

We simulated the motion of the dynamic system. This section carries out comparative dynamic analysis. As we can follow the motion of the global economy, it is straightforward to provide transitory and long term effects of changes in any parameter on the global economy. It is important to ask questions such as how a change in one country's conditions affects the national economy and global economies. First, we introduce a variable $\bar{\Delta} x(t)$ to stand for the change rate of the variable $x(t)$ in percentage due to changes in the parameter value.

A rise in the total factor productivity of country 1's industrial sector

It has been argued that productivity differences explain much of the variation in incomes across countries, and technology plays a key role in determining productivity. We now study effects of an improvement of productivity in country $1's$ industrial sector. We allow the total factor productivity to be changed as follows $A_{1i} : 1.2 \Rightarrow 1.25$. The results are plotted in Figure 2. As the system variables interact in nonlinearly, it is tedious to interpret why variables vary over time in a clear manner, even though it is not difficult to see by observing the motions in the plots. The improvement in country $1's$ industrial sector results in expansion of the sector in the country. More output is produced and more labor and capital inputs are employed. The output level and labor and capital inputs of country

$2's$ industrial sector are reduced over time. The global and country $1's$ total incomes are enhanced. Country $2's$ total income initially falls and rises in the long term. The global wealth falls initially and rises in the long term. Country $1's$ wealth and capital employed fall initially and rise in the long term. Country $2's$ capital employed is slightly affected and wealth falls initially and rises slightly in the long term. The wage rate in country 1 rises and the wage rate in country 2 falls and varies slightly in the long term. The prices of country $1's$ services and global commodity are increased and the prices of country $2's$ services and global commodity are slightly affected. Country $1's$ household initially reduces the wealth and consumption levels of all goods and services and augments these variables in the long term. Country $2's$ household reduces consumption of country $1's$ global commodity and keep the wealth and consumption levels of its own country's global commodity and services almost invariant. It is worthwhile to note that the technological change has almost no impact on the other country's service sector, and though it changes the other country's industrial and global commodity sectors.

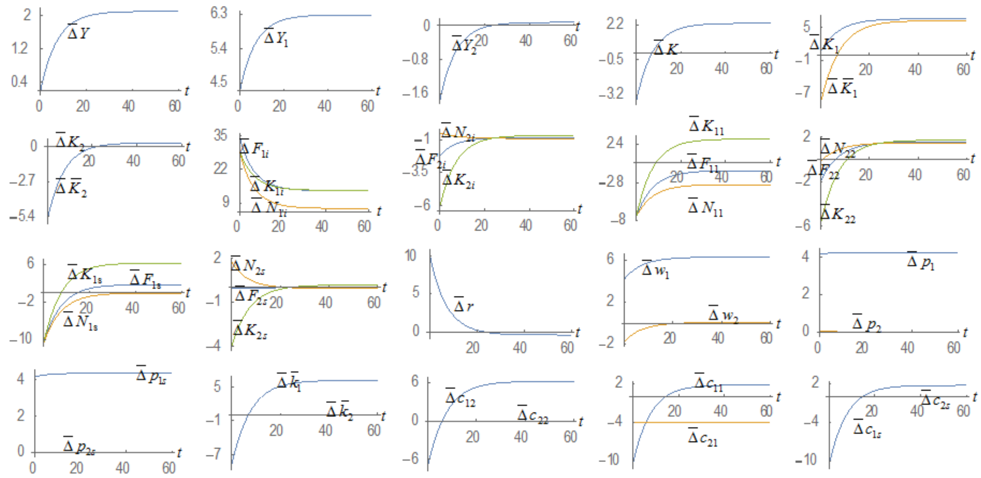

Figure no. 2 – A Rise in the Total Factor Productivity of Country 1's Industrial Sector

A fall in the output elasticity of country 1's industrial sector

This study uses the Cobb-Douglas production functions to describe production of all the sectors. The output elasticities of capital and labor represent the sector's technology and are equal to capital's and labor's shares of output. We now allow the output elasticity of country 1's industrial sector to fall as follows $\alpha_{1i} : 0.32 \Rightarrow 0.31.$ The results are plotted in Figure 3. The falling in capital share of country $1's$ industrial sector reduces the output level and capital and labor inputs of country $1's$ industrial sector. This occurs partly as a consequence that the labor is constant and immobile in the country. The parameter change increases the output level and capital and labor inputs of country $2's$ industrial sector. The global and country $1's$ total incomes are lowered. Country $2's$ total income initially rises and changes slightly in the long term. The global wealth rises initially and falls in the long term. Country $1's$ wealth and capital employed fall in the long term. Country $2's$ wealth is slightly affected and capital employed is increased. The wage rate in country 1 is reduced and the wage rate in country 2 is increased slightly. The rate of interest is reduced. The prices of country $1's$ services and global commodity are reduced and the prices of country $2's$ services and global commodity are slightly affected. Country $1's$ household initially increases the wealth and consumption levels of all goods and services and reduces these variables in the long term. Country $2's$ household increases consumption of country $1's$ global commodity and keeps the wealth and consumption levels of its own country's global commodity and services almost invariant. There are also some changes in the industrial structures. Country 1's global commodity sector produces more and employs more labor force and the sector initially employs more capital input. Country 2's global commodity sector produces less and employs less the two input factors in the long term.

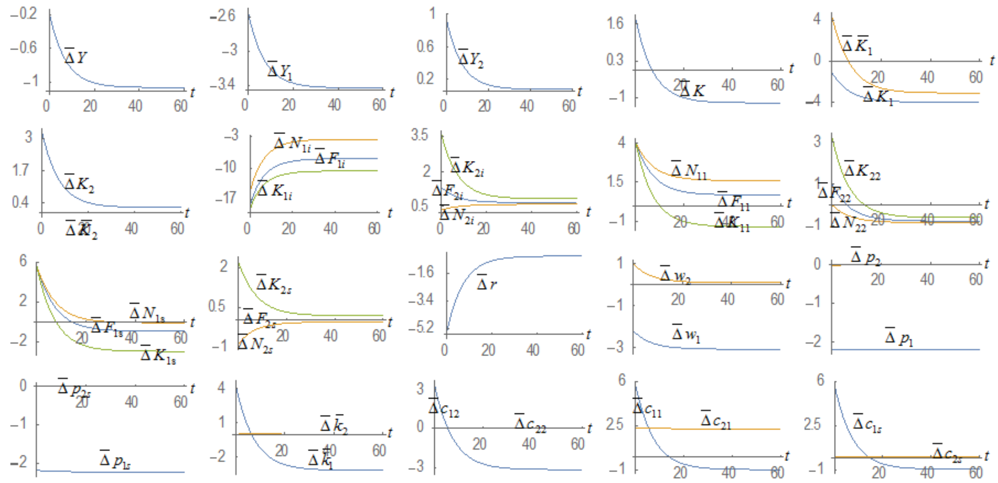

Figure no. 3 – A Fall in the Output Elasticity of Country 1's Industrial Sector

Consequences of country 1's population expansion

Chen (1992) observed: "There have been few attempts in the literature to explain long-run comparative advantage in terms of differences in initial factor endowment ratios among countries." As economists failed to analytically solve or simulate their dynamic models, it is difficult to explain transitory as well as long-run effects of initial factor endowments. As our model follows the dynamics of the global system with any conditions, it is straightforward for us to completely illustrate the effects of differences in any factor endowments. We now allow country 1's population to rise as follows $N_1 : 10 \Rightarrow 20$. The simulation results are given in Figure 4. The rise in country $1's$ population increases the global total income, wealth, and two countries' total incomes. The wage rates of the two economies are increased and rate of interest is reduced. The prices in the two economies are slightly affected. The wealth and consumption levels of all the goods and services of country $1's$ representative household are reduced in the short term and these variables are slightly affected in the long term. The wealth and consumption levels of all the goods and services of country $1's$ representative household are slightly affected. Country 1 expands the scales of the three sectors. Country 2 reduces the scale of its industrial sector and keeps the output level of services sector almost affected. Country 2 expands the scale of its global commodity sector due to the expansion of country 1's population.

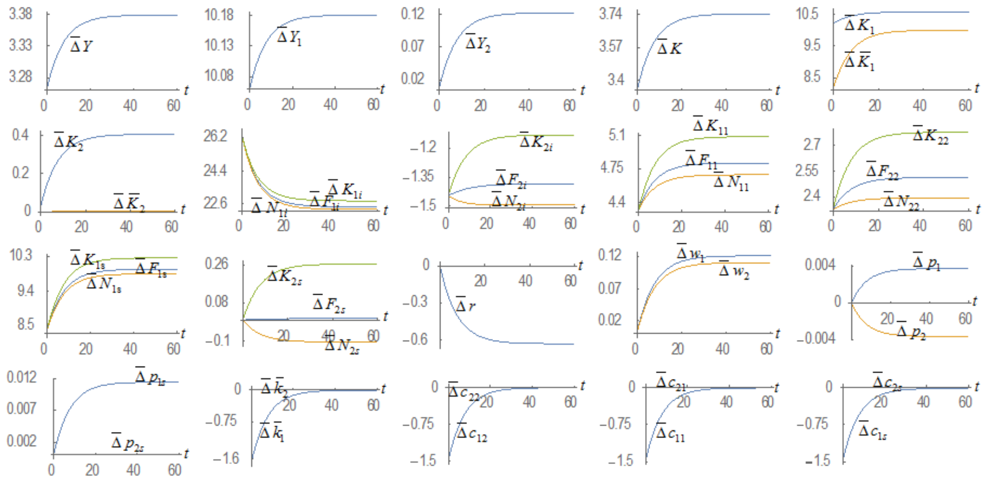

Figure no. 4 – Consequences of Country 1's Population Expansion

Country 1 increasing its propensity to consume the domestic commodity

We now study effects of the following change in country 1's propensity to consume the country's global commodity: $\xi_{110}: 0.05 \Rightarrow 0.06$. We illustrate the results in Figure 5. Country 1's household consumes more country 1's global commodity. The household has less wealth and consumes less the other global commodity and domestic services. Country 2's household has the wealth level and consumption levels of two commodities and services unchanged. Country 1's global commodity sector expands its output by employing more capital and labor force. Country 2's global commodity sector expands its output by employing more capital and labor force. A higher propensity to consume consumer goods reduces the global total income and wealth, and two countries' total incomes. Country 1 has less wealth and employs less capital stocks. Country 2 has almost same level wealth and employs less capital stocks. The increased propensity to consume goods reduces the wage rates and augments the rate of interest. The prices are slightly affected.

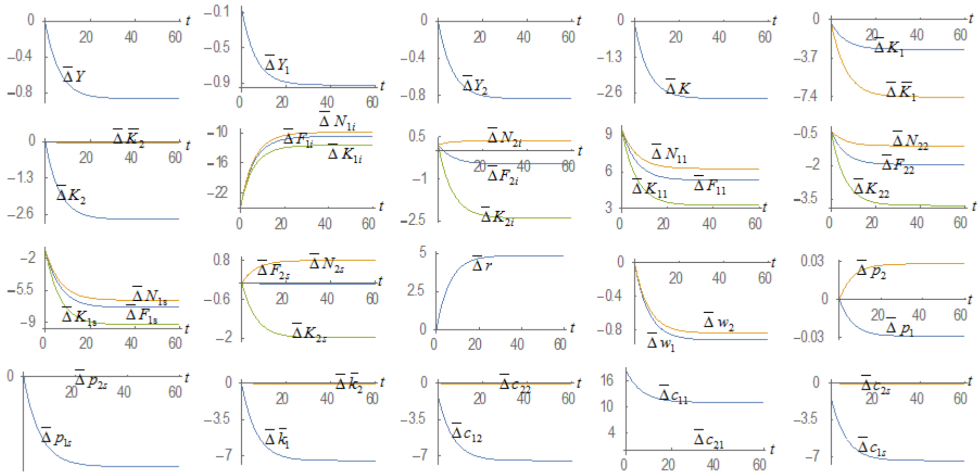

Figure no. 5 – Country 1 Increases Its Propensity to Consume the Domestic Commodity

Country 1 increasing its propensity to consume country 2's global commodity

We now study effects of the following change in country 1's propensity to consume country 2's global commodity: $\xi_{110} : 0.03 \Rightarrow 0.04.$ The results are plotted in Figure 6. Country 1's household consumes more country 2's global commodity. The household has less wealth and consumes less the domestic global commodity and services. Country 2's household has the wealth level and consumption levels of two commodities and services unchanged. The rest effects are similar to the previous case when country 1's propensity to consume country 2's global commodity is increased.

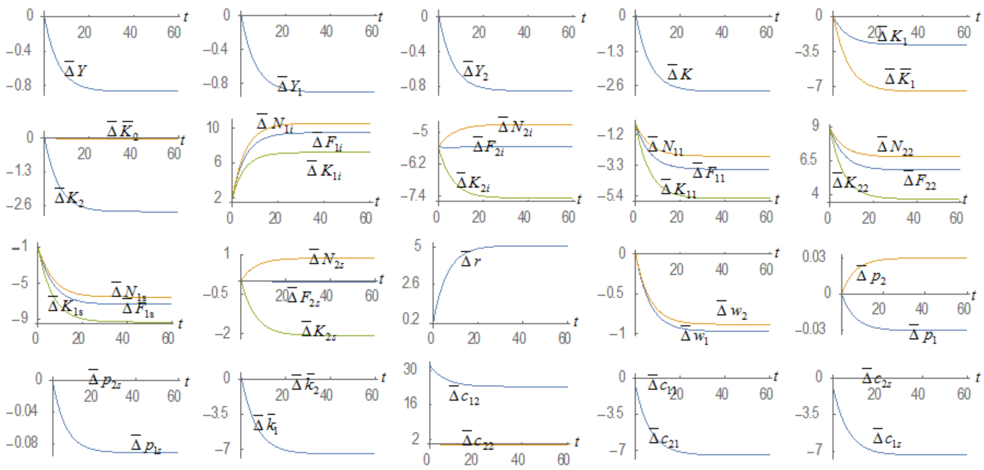

Figure no. 6 – Country 1 Increasing Its Propensity to Consume Country 2's Global Commodity

Country 1 increasing its propensity to consume services

We now study effects of the following change in country 1's propensity to consume services: $\xi_{1s0}: 0.03 \Rightarrow 0.04$. The results are plotted in Figure 7.

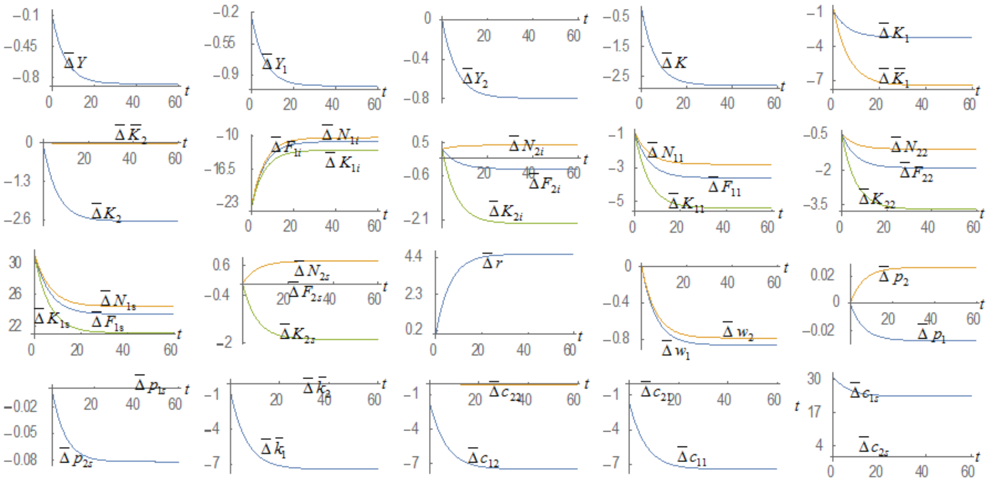

Figure no. 7 – Country 1 Increasing Its Propensity to Consume Services

Country 1 augmenting its propensity to save

We now study effects of the following change in country 1's propensity to save: $\lambda_{10}: 0.75 \Rightarrow 0.77$. We illustrate the results in Figure 8.

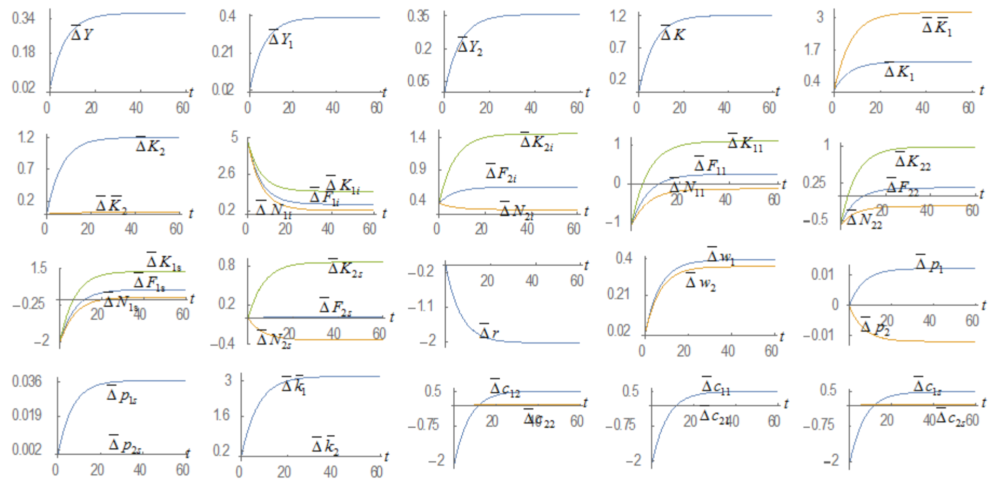

Figure no. 8 – Country 1 Augmenting Its Propensity to Save

The rise in the propensity to save augments the global total income and total wealth and the two countries' total incomes. Country 1's household increase the wealth over time.

The household initially reduces the consumption levels of two goods and services and raises these variables in the long term. The behavior of country 1's household is slightly affected. The wage rates are enhanced and the rate of interest is reduced. The prices of services are slightly increased. The price of global commodity 1 is slightly increased and the price of global commodity 2 is slightly decreased. The output levels and two inputs of the industrial sectors are increased.

5. CONCLUDING REMARKS

This paper studied the role of preferences and technological differences between two countries in determining the dynamics of capital stocks, and pattern of trade in a reformed H-O model of international trade. The paper built the trade model with endogenous wealth accumulation and labor and capital distribution between sectors and between countries under perfectly competitive markets and free trade. The model is built on the basis of the H-O model, the Solow-Uzawa neoclassical growth model and the Oniki-Uzawa trade model. The model synthesized these well-known economic models with Zhang's utility function to determine household behavior. We simulated the model for the economy to demonstrate existence of equilibrium points and motion of the dynamic system. We also examined effects of changes in output elasticity of an industrial sector, population expansion, and propensities to consume the domestic commodity, propensity to the other country's commodity, to consume services, and to hold wealth. The economic structures and interactions between different determinants of global economic growth are delicately interrelated. We might get more insights from further simulation. Our comparative dynamic analysis is limited to a few cases. The Solow model, the Uzawa two-sector growth, and the Oniki-Uzawa trade model are most well-known models in the literature of growth theory. Many limitations of our model become apparent in the light of the sophistication of the literature based on these models. We may generalize and extend our model on the basis of the traditional literature in the neoclassical growth model and trade. We may extend the model in other directions. We may introduce tariffs into the model. This study does not consider public goods and services. This study shows the role of capital accumulation as a source of economic growth and trade pattern change. There are many trade models which explicitly emphasize technological change and human capital accumulation as sources of global growth (e.g., Grossman and Helpman, 1991; Zhang, 2008).

References

Backus, D. K., and Smith, G. W., 1993. Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics, 35*(3-4), 297-316. DOI: http://dx.doi.org/10.1016/0022-1996(93)90021-O

Bajona, C., and Kehoe, T. J., 2010. Trade, growth, and convergence in a dynamic Heckscher-Ohlin model. *Review of Economic Dynamics, 13*(3), 487-513. DOI: http://dx.doi.org/10.1016/j.red.2010.05.002

Barro, R. J., and Sala-i-Martin, X., 1995. *Economic Growth*. New York: McGraw-Hill, Inc.

Baxter, M., 1992. Fiscal Policy, Specialization, and Trade in the Two-Sector Model: The Return of Ricardo? *Journal of Political Economy, 100*(4), 713-744. DOI: http://dx.doi.org/10.1086/261837

Baxter, M., Jermann, U. J., and King, R. G., 1998. Nontraded goods, nontraded factors, and international non-diversification. *Journal of International Economics, 44*(2), 211-229. DOI: http://dx.doi.org/10.1016/S0022-1996(97)00018-4

Chen, Z., 1992. Long-Run Equilibria in a Dynamic Heckscher-Ohlin Model. *The Canadian Journal of Economics, 25*(4), 923-943. DOI: http://dx.doi.org/10.2307/135772

Deardorff, A. V., and Hanson, J. A., 1978. Accumulation and a Long-Run Heckscher-Ohlin Theorem. *Economic Inquiry, 16*(2), 288-292. DOI: http://dx.doi.org/10.1111/j.1465-7295.1978.tb00288.x

Edwards, S., 1989. Temporary Terms of Trade Disturbances, the Real Exchange Rate and the Current Account. *Economica, 56*(223), 343-357. DOI: http://dx.doi.org/10.2307/2554282

Ethier, W. J., 1974. Some of the Theorems of International Trade with Many Goods and Factors. *Journal of International Economics, 4*(2), 199-206. DOI: http://dx.doi.org/10.1016/0022-1996(74)90062-2

Frenkel, J. A., and Razin, A., 1987. *Fiscal Policy and the World Economy. An Intertemporal Approach*. Cambridge, Mass.: MIT Press.

Grossman, G. M., and Helpman, E., 1991. *Innovation and Growth in the Global Economy*. Cambridge, Mass.: The MIT Press.

Heckscher, E., 1919. The Effect of Foreign Trade on the Distribution of Income. *Ekonomisk Tidskrift*, 497-512.

Heckscher, E., and Ohlin, B., 1991. *Heckscher-Ohlin Trade Theory*. Cambridge, Mass.: The MIT Press.

Ikeda, S., and Ono, Y., 1992. Macroeconomic Dynamics in a Multi-Country Economy:A Dynamic Optimization Approach. *International Economic Review, 33*(3), 629-644. DOI: http://dx.doi.org/10.2307/2527130

Lerner, A. P., 1952. Factor Prices and International Trade. *Economica, 19*(73), 1-15. DOI: http://dx.doi.org/10.2307/2549912

Mendoza, E. G., 1995. The Terms of Trade, the Real Exchange Rate, and Economic Fuctuations. *International Economic Review, 36*(1), 101-137. DOI: http://dx.doi.org/10.2307/2527429

Nishimura, K., and Shimomura, K., 2002. Trade and Indeterminacy in a Dynamic General Equilibrium Model. *Journal of Economic Theory, 105*(1), 244-260. DOI: http://dx.doi.org/10.1006/jeth.2001.2892

Ohlin, B., 1933. *Interregional and International Trade*. Cambridge: Harvard University Press.

Oniki, H., and Uzawa, H., 1965. Patterns of Trade and Investment in a Dynamic Model of International Trade. *The Review of Economic Studies, 32*(1), 15-37. DOI: http://dx.doi.org/10.2307/2296328

Pesenti, P., and Van Wincoop, E., 2002. Can Nontradables Generate Substantial Home Bias? *Journal of Money, Credit and Banking, 34*(1), 25-50. DOI: http://dx.doi.org/10.1353/mcb.2002.0034

Rogoff, K., 2002. The Purchasing Power Parity Puzzle. *Journal of Economic Literature, 34*(2), 647-668.

Rybczynski, T. M., 1955. Factor Endowments and Relative Commodity Prices. *Economica, 22*(88), 336-341. DOI: http://dx.doi.org/10.2307/2551188

Samuelson, P. A., 1948. International Trade and Equalisation of Factor Prices. *The Economic Journal, 58*(230), 163-184. DOI: http://dx.doi.org/10.2307/2225933

Samuelson, P. A., 1949. International Factor-Price Equalisation Once Again. *The Economic Journal, 59*(234), 181-197. DOI: http://dx.doi.org/10.2307/2226683

Solow, R. M., 1956. A Contribution to the Theory of Growth. *The Quarterly Journal of Economics, 70*(1), 65-94. DOI: http://dx.doi.org/10.2307/1884513

Sorger, G., 2003. On the Multi-Country Version of the Solow-Swan Model. *The Japanese Economic Review, 54*(2), 146-164. DOI: http://dx.doi.org/10.1111/1468-5876.00251

Stiglitz, J. E., 1970. Factor Price Equalization in a Dynamic Economy. *Journal of Political Economy, 78*(3), 456-488. DOI: http://dx.doi.org/10.1086/259644

Stockman, A. C., and Dellas, H., 1989. International Portfolio Nondiversification and Exchange Rate Variability. *Journal of International Economics, 26*(3-4), 271-289. DOI: http://dx.doi.org/10.1016/0022-1996(89)90004-4

Stockman, A. C., and Tesar, L. L., 1995. Tastes and Technology in a Two-country Model of the Business Cycle: Explaining International Comovements. *The American Economic Review, 85*(1), 168-185.

Stolper, W., and Samuelson, P. A., 1941. Protection and Real Wages. *The Review of Economic Studies, 9*(1), 58-73. DOI: http://dx.doi.org/10.2307/2967638
Stulz, R. M., 1987. An Equilibrium Model of Exchange Rate Determination and Asset Pricing with Nontraded Goods and Imperfect Information. *Journal of Political Economy, 95*(5), 1024-1040. DOI: http://dx.doi.org/10.1086/261500
Ventura, J., 1997. Growth and Interdependence. *The Quarterly Journal of Economics, 112*(1), 57-84. DOI: http://dx.doi.org/10.1162/003355397555127
Zhang, W. B., 1993. A woman's labor participation and economic growth: Creativity, knowledge utilization and family preference. *Economics Letters, 42*(1), 105-110. DOI: http://dx.doi.org/10.1016/0165-1765(93)90181-B
Zhang, W. B., 2008. *International Trade Theory: Capital, Knowledge, Economic Structure, Money and Prices over Time and Space*. Berlin: Springer.
Zhang, W. B., 2012. Global effects of national fiscal policies in a multi-country growth model with capital accumulation. *International Journal of Business and Globalisation, 9*(1), 42-58. DOI: http://dx.doi.org/10.1504/IJBG.2012.047521
Zhang, W. B., 2014. Tradable, Non-tradable and Education Sectors in a Multi-Country Economic Growth Model with Endogenous Wealth and Human Capital. *The Economic Research Guardian, 4*(2), 121-141.
Zhao, Y., Guo, S., and Liu, X. F., 2014. Trading frictions and consumption-output comovement. *Journal of Macroeconomics, 42*, 229-240. DOI: http://dx.doi.org/10.1016/j.jmacro.2014.08.003

APPENDIX: PROVING THE LEMMA

We now derive dynamic equations for global economic growth. From equations (2)-(4), we have

$$z_j \equiv \frac{r + \delta_{jk}}{w_j} = \frac{\bar{\alpha}_{ji} N_{ji}}{K_{ji}} = \frac{\bar{\alpha}_{jj} N_{jj}}{K_{jj}} = \frac{\bar{\alpha}_{js} N_{js}}{K_{js}}, \tag{A1}$$

where $\bar{\alpha}_{jq} \equiv \alpha_{jq} / \beta_{jq}$. From (1), (2), and (A1), we have

$$r(z_j) = \frac{\alpha_{ji} A_{ji} z_j^{\beta_{ji}}}{\bar{\alpha}_{ji}^{\beta_{ji}}} - \delta_{jk}, \quad w_j(z_j) = \frac{r + \delta_{jk}}{z_j}. \tag{A2}$$

From (A2) we have

$$z_2 = \left(\frac{r(z_1) + \delta_{2k}}{\alpha_{2i} A_{2i}} \right)^{1/\beta_{2i}} \bar{\alpha}_{2i}. \tag{A3}$$

From (1), (3) and (A1)

$$p_j = \frac{w_j z_j^{\alpha_{jj}}}{\beta_{jj} A_{jj} \bar{\alpha}_{jj}^{\alpha_{jj}}}. \tag{A4}$$

From (1), (4) and (A1)

$$p_{js} = \frac{w_j \, z_j^{\alpha_{js}}}{\beta_{js} \, A_{js} \, \overline{\alpha}_{js}^{\alpha_{js}}}. \tag{A5}$$

From (10) and (A1) we have

$$\overline{\alpha}_{ji} \, N_{ji} + \overline{\alpha}_{jj} \, N_{jj} + \overline{\alpha}_{js} \, N_{js} = z_j \, K_j . \tag{A6}$$

From (12) and (8) we have

$$N_{js} = \tilde{n}_j \, \bar{k}_j + \beta_{js} \, \xi_{js} \, N_j , \tag{A7}$$

where

$$\tilde{n}_j \equiv \left(\frac{1+r}{w_j} \right) \beta_{js} \, \xi_{js} \, N_j .$$

Insert (A7) in (11)

$$N_{ji} = \bar{n}_j - \tilde{n}_j \, \bar{k}_j - N_{jj} , \quad j = 1, 2, \tag{A8}$$

where $\bar{n}_j \equiv (1 - \beta_{js} \, \xi_{js}) N_j$. Insert (A7) and (A8) in (A6)

$$N_{jj} = \left[z_j \, K_j - \overline{\alpha}_{js} \, \beta_{js} \, \xi_{js} \, N_j - \overline{\alpha}_{ji} \, \bar{n}_j - (\overline{\alpha}_{js} - \overline{\alpha}_{ji}) \tilde{n}_j \, \bar{k}_j \right] \frac{1}{(\overline{\alpha}_{jj} - \overline{\alpha}_{ji})}. \tag{A9}$$

Insert (8) in (13)

$$\xi_{1j} \, \hat{y}_1 \, N_1 + \xi_{2j} \, \hat{y}_2 \, N_2 = p_j \, F_{jj} , \quad j = 1, 2. \tag{A10}$$

Insert (3) in (A10)

$$N_{jj} = \frac{\beta_{jj} \, \xi_{1j} \, \hat{y}_1 \, N_1}{w_j} + \frac{\beta_{jj} \xi_{2j} \, \hat{y}_2 \, N_2}{w_j}, \quad j = 1, 2. \tag{A11}$$

Insert (5) in (A11)

$$\tilde{w}_{1j} \, \bar{k}_1 + \tilde{w}_{2j} \, \bar{k}_2 + \tilde{w}_j = N_{jj} , \quad j = 1, 2. \tag{A12}$$

where

$$\tilde{w}_{1j} \equiv \frac{(1+r)\beta_{jj}\,\xi_{1j}\,N_1}{w_j},\quad \tilde{w}_{2j} \equiv \frac{(1+r)\beta_{jj}\xi_{2j}\,N_2}{w_j},\quad \tilde{w}_j \equiv \frac{w_1\,\beta_{jj}\,\xi_{1j}\,N_1}{w_j} + \frac{w_2\,\beta_{jj}\xi_{2j}\,N_2}{w_j}.$$

Equal (A12) and (A9)

$$\begin{gathered}
\left[(\bar{\alpha}_{11} - \bar{\alpha}_{1i})\tilde{w}_{11} + (\bar{\alpha}_{1s} - \bar{\alpha}_{1i})\tilde{n}_1\right]\frac{\bar{k}_1}{z_1} + (\bar{\alpha}_{11} - \bar{\alpha}_{1i})\frac{\tilde{w}_{21}}{z_1}\bar{k}_2 + \\
\frac{(\bar{\alpha}_{11} - \bar{\alpha}_{1i})\tilde{w}_1 + \bar{\alpha}_{1s}\,\beta_{1s}\,\xi_{1s}\,N_1 + \bar{\alpha}_{1i}\,\bar{n}_1}{z_1} = K_1, \\
(\bar{\alpha}_{22} - \bar{\alpha}_{2i})\frac{\tilde{w}_{12}}{z_2}\bar{k}_1 + \left[(\bar{\alpha}_{22} - \bar{\alpha}_{2i})\tilde{w}_{22} + (\bar{\alpha}_{2s} - \bar{\alpha}_{2i})\tilde{n}_2\right]\frac{\bar{k}_2}{z_2} + \\
\frac{(\bar{\alpha}_{22} - \bar{\alpha}_{2i})\tilde{w}_2 + \bar{\alpha}_{2s}\,\beta_{2s}\,\xi_{2s}\,N_2 + \bar{\alpha}_{2i}\,\bar{n}_2}{z_2} = K_2.
\end{gathered} \tag{A13}$$

Add the two equations in (A13)

$$a_1\,\bar{k}_1 + a_2\,\bar{k}_2 + a_0 = K, \tag{A14}$$

where

$$a_1 \equiv \left[(\bar{\alpha}_{11} - \bar{\alpha}_{1i})\tilde{w}_{11} + (\bar{\alpha}_{1s} - \bar{\alpha}_{1i})\tilde{n}_1\right]\frac{1}{z_1} + (\bar{\alpha}_{22} - \bar{\alpha}_{2i})\frac{\tilde{w}_{12}}{z_2},$$

$$a_2 \equiv (\bar{\alpha}_{11} - \bar{\alpha}_{1i})\frac{\tilde{w}_{21}}{z_1} + \left[(\bar{\alpha}_{22} - \bar{\alpha}_{2i})\tilde{w}_{22} + (\bar{\alpha}_{2s} - \bar{\alpha}_{2i})\tilde{n}_2\right]\frac{1}{z_2},$$

$$a_0 \equiv \frac{(\bar{\alpha}_{11} - \bar{\alpha}_{1i})\tilde{w}_1 + \bar{\alpha}_{1s}\,\beta_{1s}\,\xi_{1s}\,N_1 + \bar{\alpha}_{1i}\,\bar{n}_1}{z_1} + \frac{(\bar{\alpha}_{22} - \bar{\alpha}_{2i})\tilde{w}_2 + \bar{\alpha}_{2s}\,\beta_{2s}\,\xi_{2s}\,N_2 + \bar{\alpha}_{2i}\,\bar{n}_2}{z_2}.$$

From (15) and (A14) we solve

$$\bar{k}_1 = \phi(z_1, \bar{k}_2) \equiv \left(\frac{N_2 - a_2}{a_1 - N_1}\right)\bar{k}_2 - \frac{a_0}{a_1 - N_1}. \tag{A15}$$

It is straightforward to confirm that all the variables can be expressed as functions of z_1 and $\bar{k}_2$ by the following procedure: r by (A2) → w_j by (A2) → z_2 by (A3) → p_j by (A4) → p_{js} by (A5) → $\bar{k}_1$ by (A7) → K by (A14) → N_{jj} by (A12) → N_{ji} by (A8) → K_{js} by (A7) → F_{jq} by (1) → $\hat{y}_j$ by (A5) → K_j by (10) → c_{j1}, c_{j2} and s_j by (8) → K_{ji}, K_{jj}, K_{js} by (A1). From this procedure and (9), we have

$$\dot{\bar{k}}_1 = \tilde{\Lambda}_0(z_1, \bar{k}_2) \equiv s_1 - \phi, \tag{A16}$$

$$\dot{\bar{k}}_2 = \tilde{\Lambda}_2(z_1, \bar{k}_2) \equiv s_2 - \bar{k}_2. \tag{A17}$$

Here, we don't provide explicit expressions of the functions as they are tedious. Taking derivatives of equation (A15) with respect to t yields

$$\dot{\bar{k}}_1 = \frac{\partial \phi}{\partial z_1} \dot{z}_1 + \frac{\partial \phi}{\partial \bar{k}_2} \tilde{\Lambda}_2 , \tag{A18}$$

where we use (A17). Equal (A16) and (A18)

$$\dot{z}_1 = \tilde{\Lambda}_1\left(z_1 , \bar{k}_2\right) \equiv \left(\tilde{\Lambda}_0 - \frac{\partial \phi}{\partial \bar{k}_2} \tilde{\Lambda}_2\right)\left(\frac{\partial \phi}{\partial z_1}\right)^{-1} . \tag{A19}$$

In summary, we proved the lemma.