VARIABLES AGGREGATION-SOURCE OF UNCERTAINTY IN FORECASTING

Mihaela BRATU (SIMIONESCU)
Faculty of Cybernetics, Statistics and Economic Informatics
Academy of Economic Studies
Bucharest, Romania
mihaela_mb1@yahoo.com

Abstract

The GDP forecasting presents a particularity resulted from the fact that this macroeconomic indicator can be analyzed in its quality of aggregate. Therefore, the GDP can be predicted directly using an econometric model with lagged variables represented by the aggregate component. On the other hand, the same GDP can be predicted by aggregating the forecasts of its components. The aim of this study is to find out which strategy generates the most accurate one-step-ahead prediction and if combined forecasts can be a solution of improving the forecasts accuracy. Starting from the GDP one-year-ahead predictions made for 2009-2011 using the two strategies, measures of accuracy were calculated and the directly predicted GDP are better than those based on aggregating the components using constant and variable weights. Combined forecasts did not improve the accuracy of the predictions based on the mentioned strategies. This research is a good proof for putting the basis of considering the variables aggregation as an important source of uncertainty in forecasting.

Keywords: source of uncertainty, forecasts, accuracy, disaggregation over variables, strategy of prediction, Diebold Mariano test, combined forecasts

JEL classification: E21, E27, C51, C53

1. INTRODUCTION

One of the sources of forecast uncertainty less depth in the literature is the aggregation of variables that compose the indicator that will be forecasted. Interestingly, no author identifies this source together with other sources of uncertainty of forecasts that are based on models. In literature there are studies where the forecasts accuracy is evaluated when the interest variable are modeled using the components. In these studies the variables are also forecasted by aggregating the forecasts of its components.

The forecasts of macroeconomic aggregates are of interest not only for government, but also for private sector. The accuracy can be improved for forecasts obtained by forecasting aggregate’s components, followed by the aggregation of these predictions. The conclusion was stated in literature, but it remains valid only in the context of knowledge of data series used to draw up estimates of the models. Hubrich (2005, p. 119) showed that the aggregation of forecasts components does not necessarily help in annually forecasting.
2. LITERATURE

There are various uncertainty sources, Vega (2003, p. 18) recalling the measurement of errors, structural changes in the economy, the uncertainty that is intrinsically generated by the model, subjective adjustments of the models, the exogenous variables. Ericsson (2001, p. 68) considers that the uncertainty sources are: the forecasted variable, the economic process, based on available data, the model type used to develop forecasts, forecast horizon length.

Clements et al. (1995, p. 127) identify five sources of uncertainty for predictions based on model:
- the inaccuracy of parameter estimates;
- the Incorrect specification of the model;
- the errors in data measurement;
- the future structural changes in the economy;
- the future shocks.

Clements et al. (1995, p. 135) show that structural breaks (the slope or the level breaks) of the data series are a factor of based on model uncertainty forecasts growth.

Lanser et al. (2008, p. 3) identify four sources of uncertainty of forecasts that are based on models:
- The uncertainty in the data provided by the institution that collected them;
- The uncertainty in the series of exogenous variables;
- The uncertainty in the parameters of behavioral equations;
- The uncertainty in error terms.

Lanser et al. (2008, p. 5) modeled the four sources of uncertainty first theoretically, for each model specifying the corresponding disturbance by probability density. After the theoretical presentation, the authors assess the sources of uncertainty for Saffier model, the quarterly macroeconomic model of the Dutch Bureau for Economic Policy Analysis. This institution assessed since 1991 the quality of its macroeconomic forecasts based on simulations, producing many works about the exogenous variables, parameters and error models uncertainty.

Hendry et al. (2003, p. 6) consider that one of the causes of forecast failure is the inconsistency of parameters generated by the use of disaggregated data in the absence of structural shocks. Therefore, the aggregation / disaggregation of variables can be considered as a source of forecast uncertainty.

In the last years, due to the aggregation of geographical areas, the problem of calculating and forecasting the aggregate indicators was put for each region or member state in case of the Euro zone.

Hendry et al. (2003, p. 8) propose instead of the forecasting of an aggregate’s components, followed by the forecasts aggregation, to include in a model the variables that compose the aggregate, because the forecasts would be more accurate.

Clements et al. (2010, p. 4) lists the authors as Espasa, Senra and Albacete, Hubrich and Benalal, Diaz del Hoyo, Land, Rome and Skudelny with important contributions to preview inflation in the euro area. Fair and Shiller performed an analysis similar but for the U.S. GDP. About aggregation and disaggregation in related activity forecasting few authors have important contributions, being recalled by Hendry and Hubrich: Grunfeld and Griliches, Kohn, Lutkepohl, Pesaran, Pierce and Kumar, Van Garderen, Lee and Pesaran. Granger puts the issue of aggregation from the time series variables and Lutkepohl takes into account aggregate forecasts based on VARMA models. The concept of predictability used by Hen-
dry and Hubrich refers to the connection between variables analyzed and the appropriate data set and was previously used by Diebold and Kilian. Clements and Hendry and Hendry and Hubrich are concerned with the assessment obtained by aggregating indicators forecast accuracy of other variables. The data used by them refers to the rate of inflation in the euro area and U.S.

3. FORECASTING STRATEGIES AND THE ASSESSMENT OF PREDICTIONS ACCURACY

Clements et al. (2010, p. 25) specify two forecasting strategies: aggregating forecasts for disaggregate components and direct forecasting of the aggregates.

First, we assess the modification effects of the information set by adding the aggregates of the analyzed macroeconomic indicator. Lack of predictability depends on available information. We consider the variable over which predictions are made having an evolution as: $x_t = f_t(I_{t-1}) + u_t$. In this case, $u_t$ is a non-degenerate and unpredictable vector of random variables in relation to the information set from the past ($I_{t-1}$). By reducing the information set from $I_{t-1}$ to $J_{t-1}$ forecasts with a lower degree of accuracy will result, even if they remain unbiased, as Hendry et al. (2003, p. 8) showed. So, a larger set of information is preferred to be used in order to improve the accuracy. If we start from the conditioned distribution $(D_{T+1}^a(x_{T+1}^d / \cdot))$, the amount of information from the original set increases by disaggregating the variable $x_T^a$ in variables $x_{i,T}^a$ and by adding the aggregates in the set of information. Practically, the new set of information, having a longer length, is $J_{t-1}$, including the old aggregates and disaggregates. It is considered a scalar $x_T^a$ on which the forecasts are made and this will be split into the following form: $x_T^a = x_{1,T+1}^a \cdot g_{1,T+1} + x_{2,T+1}^a \cdot g_{2,T+1}$ (a) $g_{1,T+1} \cdot g_{2,T+1}$ - Specific weights or ponders which can be fix or may change in time. We know that: $g_{1,T+1} = 1 - g_{2,T+1}$

Assuming that GDP is the aggregated variable, then $z_T$ is the set of variables that contains:

- Aggregate variable with lags;
- Disaggregated components;
- Other variables.

Aggregate variable and its components are represented by:

$x_t^a = z_{t-1} \cdot \delta_t + \nu_t$,

$x_{i,t} = z_{i,t-1} \cdot \gamma_{i,t} + \epsilon_{i,t}$.

Conditional expectation of each component can vary over time and it is equal to the minimum value of square error of predictors: $E_{T+1} [x_{i,T+1} / z_T] = z_T^\cdot \gamma_{i,T+1}. (b)$
(b) Introducing the relation (a) in (b) it will result:

\[
E_{T+1}[x_{T+1}^a / z_T] = \sum_{i=1}^{2} g_i z_{T+1} E_{T+1}[x_{i,T+1}^a / z_T] = \sum_{i=1}^{2} g_i z_{T+1} \hat{x}_{i,T+1} = \sum_{i=1}^{2} z_{T+1} \hat{x}_{i,T+1} = \lambda_{T+1} x_T
\]

(c) \( x_{T+1}^a \) is predicted starting from : \( z_T \) : \( E_{T+1}[x_{T+1}^a / z_T] = \hat{x}_{T+1}^a \)  (d)

The above two relations, (c) and (d) are equivalent, fact that implies the same prediction error: \( \hat{x}_{T+1}^a = \lambda_{T+1} z_T \), \( \forall z_T \Rightarrow x_{T+1} - E_{T+1}[x_{T+1}^a / z_T] = v_{T+1} \). In conclusion, the direct prediction of \( x_{T+1}^a \) components is equivalent to forecasts aggregation.

In practice, even if the coefficients of models components or the specific weights change, forecasting the aggregate directly on its components have a higher degree of accuracy than if we aggregate the forecasts components. The explanations of this situation can be related to the fact that certain components of the aggregate can be volatile or that the covariance between them provide stability to the aggregate indicator. Disaggregates can be easily predicted under of an increased stability of the models coefficients or weights. Clements et al. (2010, p. 25) concluded that the aggregation of forecasts through disaggregates is a better solution in terms of accuracy than forecasting the aggregate directly. For forecasting the aggregate it is not indicated the forecasting of its changes, but the inclusion of the lags of disaggregates, which shows that the specific weights of predictions are not necessary in order to aggregate the components forecasts.

Forecast accuracy is a large chapter in the literature aimed at assessing forecast uncertainty. There are two methods used to compare the quality of forecasts: vertical methods (for example, the mean square error of prediction) and horizontal methods (such as distance in time). A comprehensive coverage of the issue taking into account all the achievements of the literature is impossible, but we will outline some important conclusions.

To assess the forecast performance, as well as their ordering, statisticians have developed several measures of accuracy. For comparisons between the mean squared errors indicators of forecasts, Granger and Newbold proposed a statistic. Another statistic is presented by Diebold and Mariano for comparison of other quantitative measures of errors. Diebold and Mariano test proposed in 1995 a test to compare the accuracy of two forecasts under the null hypothesis that assumes no differences in accuracy. The test proposed by them was later improved by Ashley and Harvey, who developed a new statistic based on a bootstrap inference. Subsequently, Diebold and Christoffersen have developed a new way of measuring the accuracy while preserving the cointegrating relation between variables.

Armstrong et al. (1995, p. 67) showed that the purpose of measuring an error of prediction is to provide information about the distribution of errors form and they proposed to assess the prediction error using a loss function. They showed that it is not sufficient to use a single measure of accuracy.

Since the normal distribution is a poor approximation of the distribution of a low-volume data series, Harvey, Leybourne, and Newbold improved the properties of small length data series, applying some corrections: the change of DM statistics to eliminate the bias and the comparison of this statistics not with normal distribution, but with the T-Student one. Clark evaluated the power of equality forecast accuracy tests, such as modified versions of the Diebold Mariano test or those used by or Newey and West, based on Bartlett core and a determined length of data series.
In literature, there are several traditional ways of measurement, which can be ranked according to the dependence or independence of measurement scale. A complete classification is made by Hyndman et al. (2005, p. 5) in their reference study in the field, “Another Look at Measures of Forecast Accuracy”.

In practice, the most used measures of forecast error are:

- Root Mean Squared Error (RMSE)

\[ RMSE = \sqrt{\frac{1}{n} \sum_{j=1}^{n} e^2_x (T_0 + j, k)} \]

- Mean error (ME)

\[ ME = \frac{1}{n} \sum_{j=1}^{n} e_x (T_0 + j, k) \]

The sign of indicator value provides important information: if it has a positive value, then the current value of the variable was underestimated, which means too small expected average values. A negative value of the indicator shows too high expected values in average.

- Mean absolute error (MAE)

\[ MAE = \frac{1}{n} \sum_{j=1}^{n} | e_x (T_0 + j, k) | \]

These measures of accuracy have some disadvantages. For example, RMSE is affected by outliers. Armstrong et al. (2000, p. 27) stress that these measures are not independent of the unit of measurement, unless if they are expressed as percentage. These measures include average errors with different degrees of variability. The purpose of using these indicators is related to the characterization of distribution errors. U Theil’s statistic is calculated in two variants by the Australian Treasury in order to evaluate the forecasts accuracy.

The following notations are used:

- a - the registered results
- p - the predicted results
- t - reference time
- e - the error (e=a-p)
- n - number of time periods
\[
U_1 = \sqrt{\frac{\sum_{t=1}^{n} (a_t - p_t)^2}{\sum_{t=1}^{n} a_t^2 + \sum_{t=1}^{n} p_t^2}}
\]

If \( U_1 \) is closer to one, the forecast accuracy is higher.

\[
U_2 = \sqrt{\frac{\sum_{t=1}^{n-1} (p_{t+1} - a_{t+1})^2}{\sum_{t=1}^{n-1} \left( \frac{a_{t+1} - a_t}{a_t} \right)^2}}
\]

If \( U_2 = 1 \) then there are no differences in terms of accuracy between the two forecasts to compare.
If \( U_2 < 1 \) the forecast to compare has a higher degree of accuracy than the naive one.
If \( U_2 > 1 \) the forecast to compare has a lower degree of accuracy than the naive one.

4. THE ASSESSMENT OF U.S. GDP FORECAST ACCURACY USING TWO FORECASTING STRATEGIES

From FRED database (Federal Reserve Economic Database) we downloaded data on the U.S. economy for variables such as GDP, final private consumption, government consumption and investment, net exports. The indicators are expressed in constant prices (billion dollars, 100 = 2005) and the period of registration is 1995-2008. The linear regression models were developed and they are used to make forecasts. One-year-ahead forecasts are made in this research for 2009-2011.

Each of forecasts was developed in two specific versions, regarding the specific weights used to aggregate the forecasts of GDP components:
- With constant weights;
- With variable weights.

In the version with constant weights, structures of the year chosen as forecast origin, the last year in data series, are used as weights. These weights show the share of consumption, investment and government spending, net exports respectively in GDP of that year.

The evolution of components weights in GDP is described using the autoregressive moving average processes. Forecasts of weights based on these models are presented in Appendix A. The models used to make one-step-ahead forecasts were built using EViews and these are presented in Table no. 1. Using data from the period 1995-2008, models for GDP and its components were obtained and used to predict the value of indicator in 2009. Using data from 1995-2009 series models used to forecast GDP in 2010 were developed.
### Table no. 1 Models used for one-year-ahead forecasts

<table>
<thead>
<tr>
<th>Year for which the prediction is made</th>
<th>The model used for direct forecasting</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>$GDP_t = -1,354 \cdot GDP_{t-3} + 2,239 \cdot cons_{t-1} + 4,185 \cdot gi_{t-1} + 0,935 \cdot net_exp_{t-1} + e_t$</td>
</tr>
<tr>
<td>2010</td>
<td>$GDP_t = -1,259 \cdot GDP_{t-3} + 2,164 \cdot cons_{t-1} + 4,001 \cdot gi_{t-1} + 0,980 \cdot net_exp_{t-1} + e_t$</td>
</tr>
<tr>
<td>2011</td>
<td>$GDP_t = -0,903 \cdot GDP_{t-3} + 1,681 \cdot cons_{t-1} + 3,961 \cdot gi_{t-1} + 0,887 \cdot net_exp_{t-1} + e_t$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year for which the forecast is made</th>
<th>The models used to develop forecasts that will be aggregated</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>$GDP_t = 1,494 \cdot cons_{t-1} + e_{1,t}$</td>
</tr>
<tr>
<td></td>
<td>$GDP_t = 5,334 \cdot gi_{t-1} + e_{2,t}$</td>
</tr>
<tr>
<td></td>
<td>$GDP_t = -21,649 \cdot net_exp_{t-1} + e_{3,t}$</td>
</tr>
<tr>
<td>2010</td>
<td>$GDP_t = 1,487 \cdot cons_{t-1} + e_{1,t}$</td>
</tr>
<tr>
<td></td>
<td>$GDP_t = 5,327 \cdot gi_{t-1} + e_{2,t}$</td>
</tr>
<tr>
<td></td>
<td>$GDP_t = -21,991 \cdot net_exp_{t-1} + e_{3,t}$</td>
</tr>
<tr>
<td>2011</td>
<td>$GDP_t = 1,486 \cdot cons_{t-1} + e_{1,t}$</td>
</tr>
<tr>
<td></td>
<td>$GDP_t = 5,311 \cdot gi_{t-1} + e_{2,t}$</td>
</tr>
<tr>
<td></td>
<td>$GDP_t = -22,526 \cdot net_exp_{t-1} + e_{3,t}$</td>
</tr>
</tbody>
</table>

**Source:** [own calculations using EViews]

**Figure no. 1** shows the underestimation of all predictions based on the two strategies. The directly forecasted GDP values are the closest of the recorded values of GDP.
Accuracy is assessed by a relative error used in making comparisons between predictions, the percentage error: 

\[ e_r = \frac{\text{GDP}_{\text{effective}} - \text{GDP}_{\text{forecasted}}}{\text{GDP}_{\text{effective}}} \times 100. \]

The predictions based on the two strategies and some accuracy measures are presented in Table no. 2. For comparisons between predictions U1 statistic is used. For U2 statistic the comparison with the forecasts based on naive model is made.

<table>
<thead>
<tr>
<th>Year</th>
<th>Directly forecasted GDP (bil. dollars 2005) (F1)</th>
<th>Forecasted GDP by aggregating the components' forecasts (constant weights) (F2)</th>
<th>Forecasted GDP by aggregating the components' forecasts (variable weights) (F3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>13068.26</td>
<td>12562</td>
<td>11210.48</td>
</tr>
<tr>
<td>2010</td>
<td>12928.25</td>
<td>12303.36</td>
<td>10779.22</td>
</tr>
<tr>
<td>2011</td>
<td>13517.41</td>
<td>12564.41</td>
<td>11069.16</td>
</tr>
</tbody>
</table>

Table no. 2 One-step-ahead forecasts of USA GDP in 2009-2011

<table>
<thead>
<tr>
<th>Year</th>
<th>RMSE</th>
<th>MPE</th>
<th>MAPE</th>
<th>U1</th>
<th>U2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>238.507</td>
<td>1.00%</td>
<td>1.38%</td>
<td><strong>0.009007</strong></td>
<td>1.931758</td>
</tr>
<tr>
<td>2010</td>
<td>847.8847</td>
<td>6.21%</td>
<td>1.38%</td>
<td>0.032886</td>
<td>6.836386</td>
</tr>
<tr>
<td>2011</td>
<td>2299.384</td>
<td>17.16%</td>
<td>1.38%</td>
<td>0.094523</td>
<td>18.53827</td>
</tr>
</tbody>
</table>

Source: [own calculations using EViews]

Hyndman et al. (2005, p. 18) showed that the percentage error can be used to calculate several indicators, including mean absolute percentage error-MAPE. For one-step-ahead forecasts made on the horizon 2009-2011, the smallest mean absolute square error registers the GDP forecasts obtained directly with only 1% for MAPE. The root mean squared error...
is the lowest for these predictions, while the U1 statistic used in comparisons shows that the one-year-ahead predictions for directly forecasted GDP are indeed the most accurate. However, all the forecasts made in this study are worse than the naïve predictions.

A generalization of Diebold-Mariano test (DM) is used to determine whether the mean squared errors matrix trace of the model with aggregation variables is significantly lower than that of the model in which the aggregation of forecasts is done. If the mean squared errors matrix determinant is used, the Diebold Mariano test can not be used in this version, because the difference between the two models MSFE determinants can not be written as an average. In this case, a test that uses a bootstrap method is recommended. The DM statistic is calculated as:

\[ DM_i = \frac{\sqrt{T} \cdot [\text{tr}(\text{MSFE}_{\text{aggregated, forecasted, model}})_h - \text{tr}(\text{MSFE}_{\text{aggregated, model}})_h]}{s} \]

\[ = \frac{1}{s} \cdot \sqrt{T} \cdot \left[ \frac{1}{T} \sum_{t=1}^{T} (em_{i,h,t} - er_{1,h,t}^2 - er_{2,h,t}^2 - er_{3,h,t}^2) \right] \]

- \( T \)-number of years for which forecasts are developed
- \( em_{i,h,t} \) - the \( h \)-steps-ahead forecast error of variable \( i \) at time \( t \) for the aggregated model
- \( er_{i,h,t} \) - the \( h \)-steps-ahead forecast error of variable \( i \) at time \( t \) for the model with aggregated forecasts
- \( s \) - the square root of a consistent estimator of the limiting variance of the numerator

The null hypothesis of the test refers to the same accuracy of forecasts. Under this assumption and taking into account the usual conditions of central limit theorem for weakly correlated processes, Diebold Mariano (DM) statistic follows a standard normal asymptotic distribution. For the variance the Newey-West estimator with the corresponding lag truncation parameter set to \( h - 1 \) is used.

The Diebold Mariano test was applied both for the version with constant specific weights of GDP components and for the one with variable weights for one-step-ahead forecasts. In the first case, the value of DM statistic (27.83) is higher than the critical one, so if we use constant weights in the forecasts aggregation model we get a better accuracy than in the case on directly forecasted GDP. If we use variable weights, the Diebold Mariano statistic value (38.23845) is greater than the critical value, so the accuracy of direct forecasts differs significantly from the one obtained by aggregating the forecasts with variable weights. The forecasts based on aggregated model have a higher degree of accuracy than those obtained by aggregating the forecast with variable specific weights.

Combined forecasts are another technique used to improve the forecasts accuracy. Therefore, we try to check this hypothesis for the three forecasts based on the mentioned strategies.

We refer to the most used combination approaches:
- optimal combination (OPT), with weak results according to Timmermann (2006, p. 9);
- equal-weights-scheme (EW);
- inverse MSE weighting scheme (INV).

Bates et al. (1969, p. 17) considered two predictions \( p1;\tau \) and \( p2;\tau \), for the
same variable $X_t$, derived $h$ periods ago. If the forecasts are unbiased, the error is calculated as: $e_{i,t} = X_{i,t} - p_{i,t}$. The errors follow a normal distribution of parameters $0$ and $\sigma_i^2$. If $\rho$ is the correlation between the errors, then their covariance is $\sigma_{12} = \rho \cdot \sigma_1 \cdot \sigma_2$. The linear combination of the two predictions is a weighted age: $c_t = m \cdot p_{1t} + (1-m) \cdot p_{2t}$. The error of the combined forecast is: $e_{c,t} = m \cdot e_{1t} + (1-m) \cdot e_{2t}$. The mean of the combined forecast is zero and the variance is:

$$\sigma_c^2 = m^2 \cdot \sigma_1^2 + (1-m)^2 \cdot \sigma_2^2 + 2 \cdot m \cdot (1-m) \cdot \sigma_{12}.$$  

By minimizing the error variance, the optimal value for $m$ is determined ($m_{opt}$):

$$m_{opt} = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2 \cdot \sigma_{12}}.$$  

The individual forecasts are inversely weighted to their relative mean squared forecast error (MSE) resulting INV. In this case, the inverse weight ($m_{inv}$) is:

$$m_{inv} = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}.$$  

Equally weighted combined forecasts (EW) are gotten when the same weights are given to all models.

The one-step-ahead combined forecasts for 2009-2011 and some accuracy measures are presented in Table no. 3, Table no. 4 and Table no. 5. The schemes presented above are utilized in making new predictions and the accuracy of these forecasts is measured by some usual indicators.

### Table no. 3 The combined one-step-ahead predictions using OPT scheme

<table>
<thead>
<tr>
<th>Year</th>
<th>F1+F2</th>
<th>F1+F3</th>
<th>F2+F3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>571.9437</td>
<td>1815.113</td>
<td>2635.84</td>
</tr>
<tr>
<td>2010</td>
<td>1008.669</td>
<td>2408.397</td>
<td>2607.55</td>
</tr>
<tr>
<td>2011</td>
<td>854.1875</td>
<td>2213.383</td>
<td>2685.48</td>
</tr>
<tr>
<td>RMSE</td>
<td>12493.57</td>
<td>11160.3</td>
<td>10662.15</td>
</tr>
<tr>
<td>MPE</td>
<td>93.91%</td>
<td>83.88%</td>
<td>80.13%</td>
</tr>
<tr>
<td>MAPE</td>
<td>93.91%</td>
<td>82.73%</td>
<td>80.23%</td>
</tr>
<tr>
<td>U1</td>
<td>0.883774</td>
<td>0.721654</td>
<td><strong>0.668546</strong></td>
</tr>
<tr>
<td>U2</td>
<td>100.7365</td>
<td>89.98679</td>
<td>85.97006</td>
</tr>
</tbody>
</table>

**Source:** [own calculations using EViews and Excel]

For combined predictions based on OPT scheme we got the best accuracy for the forecasts based on the aggregation of components predictions with constant, respectively variable weights.
Table no. 4 The combined one-step-ahead predictions using INV scheme

<table>
<thead>
<tr>
<th>Year</th>
<th>F1+F2</th>
<th>F1+F3</th>
<th>F2+F3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>558.7413</td>
<td>1760.946</td>
<td>1036.768</td>
</tr>
<tr>
<td>2010</td>
<td>992.3729</td>
<td>2345.738</td>
<td>1534.133</td>
</tr>
<tr>
<td>2011</td>
<td>829.3348</td>
<td>2142</td>
<td>1377.7</td>
</tr>
</tbody>
</table>

RMSE 12511.7
MPE 94.04%
MAPE 94.04%
U1 0.886192
U2 100.8822

Source: [own calculations using EViews and Excel]

Combining the directly forecasted GDP with the aggregation of components forecasts with variable weights we got the highest accuracy in the category of predictions based on INV scheme.

Table no. 5 The combined one-step-ahead predictions using EQ scheme

<table>
<thead>
<tr>
<th>Year</th>
<th>F1+F2</th>
<th>F1+F3</th>
<th>F2+F3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>330.74</td>
<td>1006.5</td>
<td>1259.63</td>
</tr>
<tr>
<td>2010</td>
<td>710.945</td>
<td>1473.015</td>
<td>1785.46</td>
</tr>
<tr>
<td>2011</td>
<td>400.138</td>
<td>1147.763</td>
<td>1624.263</td>
</tr>
</tbody>
</table>

RMSE 12825.12
MPE 96.39%
MAPE 96.39%
U1 0.928458
U2 103.4016

Source: [own calculations using EViews and Excel]

When the EW scheme is applied again the combined predictions based on the aggregation strategy with constant and variable weights are the best.

In the entire category of combined forecasts, the best accuracy is given by the combination between the two weighted strategies, with constant and variable weights, when optimal scheme is applied. The lowest value for U1 is a strong argument for this. However, all the initial forecasts are better than the combined ones. The percentage error is quite large and the naïve forecasts for 2009-2011 are better than the combined predictions based on the three schemes.

5. CONCLUSIONS

After the empirical study of GDP forecasts the following conclusions resulted for the horizon 2009-2011:

- Directly forecasted GDP has the highest degree of accuracy, being a better solution than the choice of forecasts obtained by aggregating the components’ predictions with variable weights using ARMA models.
- Moreover, one-step-ahead forecasts obtained directly from the econometric model are better than the combined predictions based on the forecasts resulted from applying the two strategies.
• For forecasts of indicators resulted from aggregation the evaluation of aggregation as a source of uncertainty and the choice of most accurate forecasting strategy are recommended.

We recommend the prediction of the USA GDP in the future using an econometric model with lagged variables represented by the GDP and its components. For the last three years this procedure proved to be better than the strategy based on the aggregation of components one-step-ahead forecasts. Combining the GDP values resulted from the two strategies (direct forecasting and aggregation of forecasts with constant/variable weights) did not improve the accuracy of the original predictions.

For GDP the direct forecasting could be considered a good strategy of improving the predictions accuracy. On the other hand, the differences between the values of GDP forecasts based on different strategies show that the components forecasts aggregation is a real source of uncertainty that was not mentioned before in literature. So, after our empirical research we can strongly recommend the consideration of the variables aggregation among the sources of uncertainty in forecasting.

References


Appendix A

Models used to predict variable weights

<table>
<thead>
<tr>
<th>Period</th>
<th>Variable weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995-2007</td>
<td>( g_{Cons} = 0.73288 + 0.951 \cdot g_{ConsT-1} )</td>
</tr>
<tr>
<td></td>
<td>( g_{GI} = 0.18974 + 0.627 \cdot g_{GI_{T-1}} )</td>
</tr>
<tr>
<td></td>
<td>( g_{net_exp} = 0.05008 + 0.83 \cdot g_{net_exp_{T-1}} )</td>
</tr>
</tbody>
</table>

*Source:* [own calculations using EViews]